

RETURN TO IDAHO LIBRARY

# Argonne National Laboratory

FREE CONVECTION OF A  
LOW PRANDTL NUMBER FLUID  
IN CONTACT WITH A UNIFORMLY HEATED  
VERTICAL PLATE

by

K. S. Chang, R. G. Akins,  
L. Burris, Jr., and S. G. Bankoff

#### *LEGAL NOTICE*

*This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:*

- A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or*
- B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.*

*As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.*

*Price \$2.00 . Available from the Office of Technical Services,  
Department of Commerce, Washington 25, D.C.*

ANL-6835  
Engineering and Equipment  
(TID-4500, 31st Ed.)  
AEC Research and  
Development Report

ARGONNE NATIONAL LABORATORY  
9700 South Cass Avenue  
Argonne, Illinois 60440

FREE CONVECTION OF A LOW PRANDTL NUMBER FLUID IN  
CONTACT WITH A UNIFORMLY HEATED VERTICAL PLATE

by

K. S. Chang,\* R. G. Akins,\*\* L. Burris, Jr., and S. G. Bankoff\*

Chemical Engineering Division

Based on a Thesis Submitted to  
Northwestern University in  
Partial Fulfillment of the Requirements  
for the Master's Degree  
in Chemical Engineering

\*Now at Northwestern University

\*\*Now at Kansas State University

January 1964

Operated by The University of Chicago  
under  
Contract W-31-109-eng-38  
with the  
U. S. Atomic Energy Commission



## TABLE OF CONTENTS

|  | <u>Page</u> |
|--|-------------|
| SUMMARY . . . . .  | 5           |
| I. INTRODUCTION. . . . .   | 6           |
| II. REVIEW OF LITERATURE . . . . .   | 7           |
| III. ANALYSIS. . . . .   | 13          |
| A. Basic Equations of the Problem . . . . .  | 13          |
| B. Formulation of Perturbation Equations. . . . .                                  | 16          |
| C. Zeroth-Order Solutions. . . . .   | 17          |
| D. First-Order Solutions. . . . .  | 19          |
| IV. NUMERICAL RESULTS OF PERTURBATION SOLUTIONS..                                  | 33          |
| V. CONCLUSIONS. . . . .  | 46          |
| VI. APPENDICES   |             |
| I. Functions F and $\phi$ and Derivatives for Various Prandtl<br>Numbers . . . . . | 49          |
| II. Integration of Eq. (43) . . . . .  | 67          |
| VII. NOMENCLATURE . . . . .  | 68          |
| VIII. BIBLIOGRAPHY. . . . .  | 72          |
| IX. ACKNOWLEDGMENT . . . . .   | 77          |

## LIST OF FIGURES

| <u>No.</u> | <u>Title</u>  | <u>Page</u> |
|------------|---|-------------|
| 1.         | Plate Coordinates . . . . .   | 13          |
| 2.         | Dimensionless Velocity Distributions for Various Prandtl Numbers . . . . .        | 20          |
| 3.         | Dimensionless Temperature Distributions for Various Prandtl Numbers . . . . .     | 21          |
| 4.         | Function $f'_{00}(\eta)$ . . . . .  | 27          |
| 5.         | Function $\theta_{00}(\eta)$ . . . . .  | 28          |
| 6.         | Function $f'_m(\eta)$ for $Pr = 0.1$ . . . . .                                    | 29          |
| 7.         | Function $\theta_m(\eta)$ for $Pr = 0.1$ . . . . .                                | 30          |
| 8.         | Function $f'_m(\eta)$ for $Pr = 0.03$ . . . . .                                   | 31          |
| 9.         | Function $\theta_m(\eta)$ for $Pr = 0.03$ . . . . .                               | 32          |
| 10.        | Comparison of Velocity Profiles for $Pr = 0.1$ When $Gr_L^* = 10^7$ . . . . .     | 35          |
| 11.        | Comparison of Temperature Profiles for $Pr = 0.1$ When $Gr_L^* = 10^7$ . . . . .  | 36          |
| 12.        | Comparison of Velocity Profiles for $Pr = 0.1$ When $Gr_L^* = 10^9$ . . . . .     | 37          |
| 13.        | Comparison of Temperature Profiles for $Pr = 0.1$ When $Gr_L^* = 10^9$ . . . . .  | 38          |
| 14.        | Variation of Velocity Profile with $Gr_L^*$ at $X = \frac{1}{2}$ for $Pr = 0.1$ . | 39          |
| 15.        | Comparison of Velocity Profiles for $Pr = 0.03$ When $Gr_L^* = 10^7$ . . . . .    | 40          |
| 16.        | Comparison of Temperature Profiles for $Pr = 0.03$ When $Gr_L^* = 10^7$ . . . . . | 41          |
| 17.        | Comparison of Velocity Profiles for $Pr = 0.03$ When $Gr_L^* = 10^9$ . . . . .    | 42          |

## LIST OF FIGURES

| <u>No.</u> | <u>Title</u>   | <u>Page</u> |
|------------|--|-------------|
| 18.        | Comparison of Temperature Profiles for $\text{Pr} = 0.03$ When<br>$\text{Gr}_L^* = 10^9$ . . . . . | 43          |
| 19.        | Variation of Velocity Profile with $\text{Gr}_L^*$ at $X = \frac{1}{2}$ for $\text{Pr} = 0.03$ .   | 44          |

TABLE

| <u>No.</u> | <u>Title</u>  | <u>Page</u> |
|------------|---|-------------|
| 1.         | Values of Various Functions and Derivatives for the First-Order Perturbations . . . . . | 26          |



FREE CONVECTION OF A LOW PRANDTL NUMBER FLUID IN  
CONTACT WITH A UNIFORMLY HEATED VERTICAL PLATE

by

K. S. Chang, R. G. Akins, L. Burris, Jr., and S. G. Bankoff

SUMMARY

A first-order perturbation analysis has been made of laminar free convection of an incompressible viscous fluid from a uniformly heated vertical plate with both leading and trailing edges. The method is similar to that developed for forced flow past a horizontal flat plate by Kuo,(25) which uses Lighthill's technique(27) of coordinate perturbation, and was later applied by Yang and Jerger(56) to free convection from an isothermal vertical plate. The zeroth-order solution is the exact boundary-layer solution in terms of the similarity variable,  $\eta$ , given by Sparrow and Gregg.(47) New zeroth-order results are presented here for Prandtl numbers (Pr) of 0.03 and 0.01, which show that the heating effect is still appreciable even at  $\eta = 20$ . This makes the iterative process sensitive to the guessed initial conditions. Sparrow and Gregg's solution for Pr = 0.1 was also repeated to obtain the required accuracy for the first-order perturbation. The correction due to the first-order perturbation was then made by matching a boundary limit problem for the boundary layer to an interior limit problem for the external inviscid flow. The first-order vertical velocity was obtained from a sink-strength distribution at the plate for the potential flow, which in turn was based on the horizontal velocity component at the edge of the boundary layer. Although the leading- and trailing-edge singularities are accentuated, it is not necessary to use coordinate perturbation for the first-order corrections which appear as power series in the vertical distance variable, X, with coefficients which are functions of  $\eta$ . For Pr = 0.1 and 0.03, the first ten functions were computed for both the velocity and temperature perturbations. The corrected velocity and temperature profiles were compared with the boundary-layer solution for Pr = 0.1 and 0.03 at  $0 < X < 1$ , and at modified Grashof numbers ( $Gr_L^*$ ) of  $10^7$  and  $10^9$ . As expected, the deviations are more pronounced near the leading edge and at the lower Pr and  $Gr_L^*$ . The effect of  $Gr_L^*$  in the range from  $10^5$  to  $10^9$  was also investigated at the plate midpoint. It was deduced that the boundary-layer solution and the perturbation solution at the plate midpoint coincide asymptotically at  $Gr_L^* \sim 10^{10}$  for Pr = 0.1, and at  $10^{11}$  for Pr = 0.03, which conditions are close to the transition to turbulent flow. Extensive numerical results are presented.

## I. INTRODUCTION

Steady, two-dimensional, laminar, free convection from a vertical plate with uniform surface heat flux has received less attention than that from an isothermal plate. Very few analytical studies are available in the literature.<sup>(18,47)</sup> On the other hand, solutions for finite, semi-infinite, and infinite isothermal plates are available for a wide range of Prandtl numbers.

Although the phenomenon of laminar free convection in a fluid in contact with a vertical plate can be described by the equations of motion, energy, and continuity, as is the case for a well-defined system, intimate coupling between temperature gradient and fluid motion makes the problem more difficult to analyze than that of the other types of flow. Usually, the laminar boundary-layer theory has been applied to obtain zeroth-order solutions, which are in many cases, adequate descriptions of the physical phenomena.

However, since the boundary-layer equations for laminar free convection along a vertical plate are adequate only as the Grashof number approaches infinity, it is evident that the solutions thus obtained are not accurate for finite or moderate Grashof numbers. This deviation can be clearly seen from the comparison made by Ostrach,<sup>(34)</sup> between the experimental data of Schmidt and Beckmann,<sup>(42)</sup> and the solutions obtained by the boundary-layer theory. The effect of Grashof number, therefore, must be explicitly incorporated in the equations and in the resulting solutions.

Refinements of the boundary-layer solutions by incorporating the effect of Grashof number can be achieved by applying a perturbation method in which the velocity and temperature fields around the plate are divided into two regions (i.e., a boundary-layer region and an inviscid or potential-flow region), and in which the fields in the two regions are smoothly matched. This is an approach similar to that followed by Kuo<sup>(25)</sup> in treating two-dimensional incompressible forced flow over a horizontal flat plate at moderate Reynolds numbers. Yang and Jerger<sup>(56)</sup> applied this method to laminar free convection from an isothermal vertical plate for Prandtl numbers of 0.72 and 10, and improved Ostrach's solutions.<sup>(34)</sup>

The purpose of the present analysis is, therefore, to apply a similar perturbation theory on steady, two-dimensional, laminar free convection from a finite vertical plate having uniform surface heat flux. In the following sections, boundary-layer equations are derived as the zeroth-order approximation, and the first-order perturbation equations are formulated from the potential-flow theory. Both the boundary-layer equations and the first-order perturbation equations are then solved. Solutions to the boundary-layer equations for Prandtl numbers of 0.01 and 0.03 are newly obtained and added to the list of existing solutions for Prandtl

numbers of 0.1, 1, 10, and 100. The perturbation solutions are developed for Prandtl numbers of 0.1 and 0.03, and the results are compared with the boundary-layer solutions.

These low Prandtl numbers are chosen because: (1) recent developments in nuclear reactors have led to significant interest to liquid metals, either as coolants in reactors or as solvent media in pyrometallurgical processes for treating nuclear fuel materials; and (2) the deviations from the boundary-layer solutions are larger for liquids of low Prandtl numbers.

## II. REVIEW OF LITERATURE

Free convection for a vertical plate has been studied considerably. As pointed out in the introduction, most of the studies have been for isothermal plates, with relatively few studies for plates with uniform surface heat flux.

In 1881, L. Lorenz(29) presented, in his pioneer paper, an analysis for a heated vertical plate at uniform temperature in air at rest. In his analysis, he assumed that the temperature and velocity at any point depend only on the distance from the plate. Although oversimplified by inadequate assumptions, his analysis satisfied the then-existing experiments and revealed for the first time the complex nature of the heat-transfer coefficient by free convection.

In 1922, Griffiths and Davis(21) determined by direct measurements the distribution of velocity and temperature close to vertical plates in which the surface temperature was kept uniform.

In 1928, Nusselt and Jürges(33) improved the theory and carried out the measurement of temperature field for a vertical plate at 100°C.

In 1930, Schmidt and Beckmann(42) conducted experiments about three different vertical plates (two 12- by 25-centimeter plates, and one 50- by 50-centimeter plate), in which velocity was measured at various points along the isothermal plates by means of a quartz-filament anemometer, and the temperature was measured by means of manganese-constantan thermocouples with wires of 0.015-mm diameter. The edges of the plates were smoothed either symmetrically or nonsymmetrically. Their experiments showed that Lorenz's assumptions were invalid. Schmidt and Beckmann set up the partial differential equations for an isothermal vertical plate applying the approximations of boundary-layer theory. In 1930, E. Pohlhausen applied a similarity method he had used in 1921(36) for the boundary layer on a semi-infinite flat plate parallel to a uniform flow. His results, incorporated by Schmidt and Beckmann,(42) showed how the partial differential equation obtained by Schmidt and Beckmann could be

transformed into ordinary differential equations with a single independent variable, i.e., a similarity variable. The Prandtl number appeared as a parameter, and five boundary conditions had to be satisfied in the equations, three at the plate and two at infinity. Because of the inherent difficulties in solving the equations, Pohlhausen used the measurements by Schmidt and Beckmann of the normal gradients of temperature and velocity in air close to a heated plate to obtain two more boundary conditions at the plate. Starting with the five given boundary conditions at the plate, he obtained a solution in series of the equations for air ( $\text{Pr} = 0.733$ ). (As will be seen shortly, solutions for various Prandtl numbers have been obtained by Schuh and Ostrach.) In 1932, Schmidt<sup>(41)</sup> devised a Schlieren method to visualize the laminar thermal boundary layer around heated bodies in free convection.

Also in 1932, King<sup>(23)</sup> correlated the data of his own experiments and of experiments done by others with air, water, alcohol, and oil in contact with vertical plates and compared the heat-transfer behavior of different plate lengths.

In 1932, Kimball and King<sup>(22)</sup> reported a theoretical investigation based on observation of the locus of velocity maxima.

An experimental correlation derived in 1934 by H. Lorenz,<sup>(28)</sup> based on measurements on a vertical hot plate in oil is given by Schlichting.<sup>(40)</sup> In 1935 and 1936, Weise<sup>(54)</sup> and Saunders<sup>(37)</sup> respectively, obtained extensive data for short vertical plates, which were later correlated by McAdams.<sup>(30)</sup>

In 1939, Saunders<sup>(38)</sup> also presented approximate solutions for air and compared his results with E. Pohlhausen's<sup>(42)</sup> and Squire's.<sup>(20)</sup> He obtained some experimental data for mercury and water and appears to have been the first to have studied free convection with a liquid metal. However, the plate he used was pressed into the surface of fireclay containing a heater coil and constituted a portion of the wall surface. Therefore, it had no leading and trailing edges.

Schuh (1946)<sup>(43)</sup> extended E. Pohlhausen's calculations to high Prandtl numbers ( $\text{Pr} = 10, 100$ , and  $1000$ ) such as exist in oils. Schlichting<sup>(40)</sup> summarized the results.

McAdams<sup>(30)</sup> quotes a theoretical relation for vertical planes derived by Eckert. The derived equation correlated the data of Schmidt and Beckmann for air, but predicted much lower results than conventional relations for fluids having low Prandtl numbers. In 1948, Eckert and Soehnghen<sup>(8,9)</sup> used interferometers to study the mechanism of free convection from a heated vertical plate into air. Schlichting<sup>(40)</sup> summarized another comparison between theoretical results with measurements on heated vertical plates reported in 1951 by Eckert and Jackson.<sup>(7)</sup>

Sugawara and Michiyoshi, in 1951,(51) studied unsteady free convection on a vertical wall.

In 1953, Ostrach,(34) starting from the complete time-independent equations for variable properties, determined the conditions under which E. Pohlhausen's equations adequately describe the physical process. These equations were then solved numerically for various Prandtl numbers ( $\text{Pr} = 0.01, 0.72, 0.733, 1, 2, 10, 100, \text{ and } 1000$ ). The resulting dimensionless temperature and velocity distributions showed how the Prandtl number affects the shape of the curves and the relative thickness of the thermal and velocity boundary layers. Ostrach compared his solutions with the experimental data of Schmidt and Beckmann and pointed out that, in general, the agreement was good for small values of the similarity variable, less satisfactory for larger values of the similarity variable.

All the approaches so far mentioned were for isothermal vertical plates or planes. In 1956, Sparrow and Gregg(47) were the first to analyze laminar free convection for a vertical plate with uniform surface heat flux. They transformed the original differential equations to ordinary differential equations by introducing a new similarity variable based on the heat flux at the surface of the plate. These equations were solved numerically for several Prandtl numbers ( $\text{Pr} = 0.1, 1, 10, \text{ and } 100$ ), and in 1954 Dotson's<sup>(6)</sup> experimental data for air were used to verify the theory. The solutions were compared with the results obtained by the von Kármán-K. Pohlhausen method. This is the only available rigorous analytical study made for a vertical plate having uniform surface heat flux.

In 1958, Siegel(44) studied the transient behavior of free convection for a vertical flat plate of infinite width and semi-infinite length.

Sparrow and Gregg (1958)(48) analyzed laminar free convection from a vertical flat plate having a nonuniform surface temperature and obtained numerical solutions for Prandtl numbers of 0.7 and 1.0.

In 1956, Finston(12) had showed that, according to the boundary-layer theory, the problem of free convection past a vertical plate has an exact solution for a plate temperature which is proportional to a power of the distance from the leading edge of the plate. In 1958, Foote<sup>(13)</sup> extended Finston's method and obtained the solution by asymptotic expansions. Numan and Pohlhausen (1958)(32) numerically evaluated Finston's parametric equations for air ( $\text{Pr} = 0.733$ ).

Sparrow and Gregg (1958)(49) analyzed the variable fluid-property problem for laminar free convection from an isothermal vertical flat plate in which constant-property analysis and reference-temperature relations for gases and liquid mercury were given in detail.

In 1959, Fujii<sup>(14)</sup> applied a modified integral method for laminar free convection from a vertical flat surface and supplemented Squire's approximation as referenced by Goldstein.<sup>(20)</sup> He also attempted nonisothermal surface problems. In addition, he analyzed turbulent free convection from a vertical surface.<sup>(15)</sup>

In 1959, Bobco<sup>(4)</sup> obtained an approximate closed-form solution to the problem of laminar free convection on a vertical plate with the nonuniform wall heat flux prescribed by Sparrow.<sup>(48)</sup>

Kutateladze et al., (1959)<sup>(26)</sup> reviewed heat transfer for free convection in liquid metals.

In 1960, Goldstein and Eckert,<sup>(19)</sup> using a Zehnder-Mach interferometer, studied the steady and transient-free convection boundary layer about a uniformly heated vertical plate in water.

Yang (1960)<sup>(55)</sup> established the necessary and sufficient conditions required for the existence of similarity solutions to the problem of steady and unsteady free convection on vertical plates with various surface temperature distributions.

Sparrow and Gregg (1960)<sup>(50)</sup> studied quasi-steady free convection for a vertical plate in a gas ( $\text{Pr} = 0.72$ ), together with the first-order deviations.

Eichhorn (1960)<sup>(10)</sup> analyzed the effects of blowing and suction on the free convection flow and heat transfer about a vertical isothermal plate. In 1961, Sparrow and Cess<sup>(46)</sup> extended Eichhorn's analysis on free convection from a vertical plate with suction or blowing to more general temperature and velocity distributions.

Bayley et al., (1961)<sup>(2)</sup> experimentally investigated free convection with a liquid metal in a thermosyphon tube.

Elshin (1961)<sup>(11)</sup> presented an approximate solution to a free-convection problem on a nonisothermal vertical plane in the form of polynomials of the fifth degree in von Kármán's correlations. Elshin compared the results for  $\text{Pr} = 1$  with those of Sparrow and Gregg.<sup>(48)</sup>

Chung and Anderson (1961)<sup>(5)</sup> analyzed the behavior of the unsteady, laminar, free-convection, boundary-layer equations for a plate parallel to the acceleration field. The first-order deviations of the velocity and temperature profiles and the heat transfer, from the quasi-steady state, were computed for  $\text{Pr} = 0.72$ .

In 1962, Klyachko<sup>(24)</sup> analyzed the transition from laminar to turbulent flow in free convection over a vertical plane.

Acrivos (1962)(1) showed how an approximate but accurate expression can be obtained for the rate of heat and mass transfer in laminar boundary-layer flows by piecing together certain exact asymptotic solutions. He pointed out that free convection is mathematically similar to forced convection if the velocity is replaced by a characteristic velocity.

Scherberg (1962)(39) investigated free convection in the neighborhood of several types of thermal leading edges on a vertical wall.

Szewczyk (1962)(52) theoretically and experimentally investigated the stability and transition of the free-convection layer along a vertical flat plate. He observed that the critical Grashof number parameter obtained experimentally was much less than the one obtained theoretically.

Young and Yang (1962)(57) applied a perturbation analysis to examine the effect of small cross-flow and surface temperature variation on boundary-layer characteristics of laminar free convection along a vertical plate.

Gebhart (1962)(18) discussed the conditions under which the effects of viscous dissipation in free convection become important, and determined the criteria for isothermal and uniform surface heat flux plates. Gebhart (1962)(17) also predicted the effect of viscosity on inflection points in free convection.

Menold and Yang (1962)(31) presented exact asymptotic solutions for unsteady free convection on a vertical plate with arbitrary surface temperature or heat flux variations and Prandtl number.

Yang and Jerger (1962)(56) formulated first-order perturbations of laminar free convection on an isothermal vertical plate, in which the calculations were made for Prandtl numbers of 0.72 and 10.0. The results were compared with the experimental data of Schmidt and Beckmann for air.

This review summarizes free-convection studies on vertical plates and surfaces associated either remotely or closely with the present analysis. Since not all these papers have direct connection with this report, those more directly related to the subject must be mentioned especially.

As was mentioned previously, Pohlhausen's solution for air ( $\text{Pr} = 0.733$ ) of the boundary-layer equations, based on the experimental data of Schmidt and Beckmann,(42) was extended by Schuh(43) to Prandtl numbers of 10, 100, and 1000. Ostrach(34) perfected the analysis of the boundary-layer equations for an isothermal vertical plate and numerically solved the equations for various Prandtl numbers of 0.01, 0.72, 0.733, 1, 2, 10, 100, and 1000. The comparison of the solutions with the detailed

experimental data of Schmidt and Beckmann for air revealed that the experimental local velocity profiles, when properly nondimensionalized, do show significant and consistent deviations from the theoretical curve of the boundary-layer theory. As Ostrach pointed out, the deviations vary according to the distance from the leading edge of the plate. This led to the refinement of solutions by Yang and Jerger(56) in which the first-order perturbations for the velocity and temperature fields have been calculated for Prandtl numbers of 0.72 and 10. The method used was similar to that applied by Kuo (1953)(25) on the forced flow of an incompressible viscous fluid past a horizontal flat plate at moderate Reynolds numbers.

For vertical plates with uniform surface heat flux, Sparrow and Gregg,(47) introducing a new similarity variable based on a modified local Grashof number, transformed the boundary-layer equations to ordinary differential equations and obtained numerical solutions for Prandtl numbers of 0.1, 1, 10, and 100. However, solutions are not available for Prandtl numbers less than 0.1 (for free convection with liquid metals). The present analysis therefore supplements the solutions for this range and refines them further by using a perturbation method.

### III. ANALYSIS

#### A. Basic Equations of the Problem

This report will treat the problem of steady, two-dimensional, laminar, free convection in an incompressible viscous fluid from a uniformly heated vertical plate, with both leading and trailing edges, by starting from the complete equations of motion, energy, and continuity. The rectangular Cartesian coordinate system is adopted as shown in Fig. 1. The origin is located at the leading edge of the plate, and the plate itself is defined by  $0 \leq x \leq L$  and  $y = 0$ . Here, the lines of force of the gravitational field are chosen to be parallel to the  $x$ -coordinate. The direction of the  $y$ -coordinate is chosen so that the system, when rotated 90 degrees clockwise, represents the conventional complex plane. The complete and exact equations of motion, energy, and continuity<sup>(3)</sup> will be given in tensor notation:

$$\rho \left( \frac{\partial \mathbf{V}_i}{\partial t} + \mathbf{V}_j \mathbf{V}_{i,j} \right) = -p_{,i} - \mathbf{T}_{i,j}^j + \rho g_i; \quad (1)$$

$$\rho C_v \left( \frac{\partial T}{\partial t} + \mathbf{V}_j T_{,j} \right) = -q_j^j - T \left( \frac{\partial p}{\partial T} \right)_v \mathbf{V}_{,j}^j - \mathbf{T}_i^j \mathbf{V}_{,j}^i; \quad (2)$$

$$\frac{\partial \rho}{\partial t} + (\rho \mathbf{V}_j)_{,j} = 0; \quad (3)$$

where  $\rho$  is density,  $t$  time,  $\mathbf{V}_j$  velocity vector,  $p$  pressure,  $\mathbf{g}_i$  body force vector per unit mass,  $C_v$  heat capacity at constant volume per unit mass,  $T$  absolute temperature,  $q_j^j$  heat flux vector,  $v$  volume per unit mass,

and  $\mathbf{T}_i^j$  component of stress tensor. Comma notation represents covariant differentiation, and the summation convention is employed. Equations (1) through (3) represent a system of five equations, and no exact solutions are possible or known. Simplification from a physical viewpoint is, therefore, necessary. For the two-dimensional steady-state case, all the time-dependent terms vanish, and the three equations represented by Eq. (1) reduce to two.

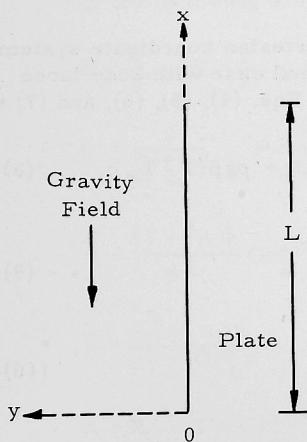


Figure 1. Plate Coordinates

In free convection, the important term is that of body force which gives buoyancy to generate free convection. Therefore,  $\rho g_i$  in Eq. (1) will be expanded in Taylor series in  $T$  about some reference temperature  $T^*$ :

$$\begin{aligned} \rho g_i &= \rho * g_i + \left( \frac{\partial \rho}{\partial T} \right)_{T^*} (T - T^*) g_i + \dots \\ &= \rho * g_i - \rho * \beta * g_i (T - T^*) + \dots \end{aligned} \quad (4)$$

where  $\rho^*$  and  $\beta^*$  represent the density and the coefficient of thermal expansion, respectively, at reference temperature. Generally speaking, density  $\rho$  is a function of pressure  $p$  and temperature  $T$ . However, the assumption of incompressible fluid rules out pressure dependency. The bulk temperature,  $T_\infty$ , far from the plate is usually taken as the reference temperature, and the transport properties are therefore evaluated at this temperature. However, when all the transport properties, including density, are assumed to be constant in Eqs. (1), (2), and (3) (except in the  $\rho g_i$  term), these properties evaluated at temperature  $T_\infty$  do not necessarily describe the physical phenomenon precisely. The question arises, therefore, as to the most appropriate reference temperature at which to evaluate the physical properties. One possibility is a reference temperature determined from a variable fluid-property analysis, such as the one by Sparrow and Gregg.<sup>(49)</sup> For simplicity, the asterisks in Eq. (4) will be removed with the understanding that all the transport properties take such constant values.

For incompressible Newtonian fluids with constant transport properties:<sup>(45)</sup>

$$\nabla_{,j}^j = 0; \quad (5)$$

$$q_{,j}^j = -k T_{,j}^j; \quad (6)$$

$$\tau_{i,j}^j = -\mu \nabla_{i,j}^j; \quad (7)$$

where  $k$  and  $\mu$  are thermal conductivity and viscosity, respectively.

With the more familiar notation of the Cartesian coordinate system, Eqs. (1), (2), and (3) for the steady two-dimensional case with body force parallel to the  $x$  direction, simplify by virtue of Eqs. (4), (5), (6), and (7) to

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \rho g + \rho g \beta (T - T_\infty); \quad (8)$$

$$\rho \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = - \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right); \quad (9)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{\mu}{C_p \rho} \Phi_v; \quad (10)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \quad (11)$$

where  $u$  and  $v$  are the velocity components in the  $x$  and  $y$  directions, respectively;  $\alpha$  is the thermal diffusivity,  $k/(C_p \rho)$  ( $C_v$  is replaced by  $C_p$ , heat capacity at constant pressure per unit mass); and  $\Phi_v$  is the dissipation function. Here,  $g_i$  is replaced by  $g$ , the gravitational acceleration, and  $g_i = -g$ .

For fluids such as liquid sodium, mercury, gases at ordinary temperatures, water, and viscous silicones, the viscous dissipation is usually negligible in laminar free convection.<sup>(18)</sup> The term  $\mu \Phi_v/(C_p \rho)$  is, therefore, removed from Eq. (10) without any further consideration.

Equation (11) implies the existence of a stream function  $\psi$  such that

$$u = \frac{\partial \psi}{\partial y} = \psi_y; \quad v = -\frac{\partial \psi}{\partial x} = -\psi_x; \quad (12)$$

where the subscripts denote partial differentiation.

The boundary conditions associated with the present problem are:

$$\left. \begin{array}{l} \text{at } y = 0 \text{ and } 0 \leq x \leq L: \quad u = v = 0, q = -k \frac{\partial T}{\partial y} \Big|_{y=0} \\ \text{at } y \rightarrow \infty: \quad u \rightarrow 0, v \rightarrow 0, T \rightarrow T_\infty, p \rightarrow p_\infty \end{array} \right\}, \quad (13)$$

where  $q$  is the heat flux at the surface of the plate, and  $p_\infty$  is the static pressure of the undisturbed environment.

If the following dimensionless quantities are introduced:

$$\bar{x} = \frac{x}{L}, \quad \bar{y} = \frac{y}{L}; \quad (14)$$

$$\left. \begin{array}{l} \psi = \frac{\psi}{Gr^* \nu}, \quad P = \frac{(p - p_\infty)L^2}{\rho \nu^2 Gr^{*4/5}}, \quad G = \frac{g L^3}{\nu^2 Gr^{*4/5}} \\ \Theta = \frac{(T - T_\infty)k}{q L} Gr_L^{*1/5}, \quad Gr_L^* = \frac{g \beta L^4 q}{k \nu^2} \end{array} \right\} \quad (15)$$

$$\bar{u} = \frac{\partial \bar{\psi}}{\partial \bar{y}} = \frac{u L}{Gr^* \nu}, \quad \bar{v} = -\frac{\partial \bar{\psi}}{\partial \bar{x}} = \frac{v L}{Gr^* \nu}; \quad (16)$$

$$X = \bar{x}, \quad Y = Gr_L^{*1/5} \bar{y}, \quad \Psi = Gr_L^{*4/5} \bar{\psi}; \quad (17)$$

where  $L$  is the height of the plate,  $\bar{\psi}$  a dimensionless stream function,  $P$  the dimensionless pressure,  $G$  the dimensionless gravitational acceleration,  $\Theta$  the dimensionless temperature,  $\nu$  the kinematic viscosity,  $\bar{u}$  and  $\bar{v}$  the dimensionless velocities, and  $Gr_L^{*}$  the modified Grashof number based on  $L$ . Then Eqs. (8), (9), and (10) are transformed, respectively, to:

$$\Psi_Y \Psi_{XY} - \Psi_X \Psi_{YY} = - P_X + Gr_L^{*-2/5} \Psi_{XXY} + \Psi_{YYY} - G + \Theta ; \quad (18)$$

$$Gr_L^{*-2/5} (-\Psi_Y \Psi_{XX} + \Psi_X \Psi_{XY}) = - P_Y - Gr_L^{*-4/5} \Psi_{XXX} - Gr_L^{*-2/5} \Psi_{XXY} ; \quad (19)$$

$$\Psi_Y \Theta_X - \Psi_X \Theta_Y = Pr^{-1} (Gr_L^{*-2/5} \Theta_{XX} + \Theta_{YY}) ; \quad (20)$$

where the subscripts denote partial differentiations with respect to the variables indicated. For example,  $\Psi_{XXY} = (\partial^3 \Psi) / (\partial X^2 \partial Y)$ , and  $Pr$  is the Prandtl number of the fluid under consideration. The second transformation of the coordinate system from the  $\bar{x}$ ,  $\bar{y}$ -system to the  $X$ ,  $Y$ -system brought the order of magnitude of  $Y$  to that of  $X$ . This is based on the knowledge (from the boundary-layer solutions) that the ratio  $\bar{y}/\bar{x}$  is of the order of magnitude of  $Gr_L^{*-1/5}$ , as is seen from Sparrow and Gregg's solution.<sup>(47)</sup> The function  $\Psi$  is also introduced in such a manner that  $\Psi_Y / (-\Psi_X) = \bar{u} / (Gr_L^{*1/5} \bar{v})$ .

### B. Formulation of Perturbation Equations

To solve Eqs. (18), (19), and (20) by a perturbation method, it is assumed that the function  $\Psi$ , the pressure  $P$ , and the temperature  $\Theta$  can be expanded in a small constant parameter  $\epsilon$ :

$$\Psi = \Psi^{(0)} + \epsilon \Psi^{(1)} + \epsilon^2 \Psi^{(2)} + \dots ; \quad (21)$$

$$P = P^{(0)} + \epsilon P^{(1)} + \epsilon^2 P^{(2)} + \dots ; \quad (22)$$

$$\Theta = \Theta^{(0)} + \epsilon \Theta^{(1)} + \epsilon^2 \Theta^{(2)} + \dots . \quad (23)$$

In free convection, the fundamental boundary-layer concept is that all effects due to the velocity and temperature fields take place inside a narrow region along the solid surface outside of which these effects are ignored. As will be seen in Section D, the horizontal velocity at the edge of the boundary layer does not vanish, and this indicates that the flow exists outside of the boundary layer. The flow field exterior to the plate, therefore, may be subdivided into two regions: a boundary-layer next to the plate, and a region outside the boundary layer. This subdivision of flow field is analogous to that in the problem of an equivalent forced flow along a plate but different from it only in that the surrounding fluid is at rest in free convection.

If the viscous effects take place inside the boundary layer alone, the flow outside this layer is essentially frictionless and irrotational, and therefore can be called inviscid or potential flow. Then the problem can be divided into a "boundary limit problem" for the boundary layer, and an "interior limit problem" for the external inviscid flow. Joining a solution of the interior limit problem at  $\bar{y} = 0$  to that of a boundary limit problem at  $\bar{y} = \infty$  is well known.<sup>(35)</sup> The boundary-layer solution can then be taken as the zeroth-order approximation, and the solution matched with that of potential flow can be taken as the first-order approximation. To do this, the two fields must join smoothly. This means that the horizontal velocity properly nondimensionalized in the potential-flow field [which is  $0(\epsilon)$ , according to the series expansion in the parameter  $\epsilon$ ], must be equal to that of the viscous flow field at the edge of the boundary-layer, which is  $0(\text{Gr}_L^{1/5})$ . Therefore,  $\text{Gr}_L^{1/5}$  will be taken as the parameter  $\epsilon$  in the series expansions.

When this definition of  $\epsilon$ , together with the expansions of  $\Psi$ ,  $P$ , and  $\Theta$ , is substituted in Eqs. (18), (19), and (20), and the coefficients of like powers of  $\epsilon$  are equated, the following sets of equations result (according to the order of perturbation). However, the order greater than or equal to 2 will not be given here.

The zeroth-order perturbation equations are:

$$\frac{\Psi^{(0)}}{Y} \frac{\Psi^{(0)}_{XY}}{X} - \frac{\Psi^{(0)}_X}{X} \frac{\Psi^{(0)}_{YY}}{Y} = - \frac{P^{(0)}_X}{X} + \frac{\Psi^{(0)}_{YYY}}{Y} - G + \Theta^{(0)}; \quad (24)$$

$$- \frac{P^{(0)}_Y}{Y} = 0; \quad (25)$$

$$\frac{\Psi^{(0)}_Y}{Y} \Theta^{(0)}_X - \frac{\Psi^{(0)}_X}{X} \Theta^{(0)}_Y = \frac{1}{\text{Pr}} \Theta^{(0)}_{YY}. \quad (26)$$

The first-order perturbation equations are:

$$\frac{\Psi^{(0)}_Y}{Y} \Psi^{(1)}_{XY} + \frac{\Psi^{(1)}_Y}{Y} \Psi^{(0)}_{XY} - \frac{\Psi^{(0)}_X}{X} \Psi^{(1)}_{YY} - \frac{\Psi^{(1)}_X}{X} \Psi^{(0)}_{YY} = - \frac{P^{(1)}_X}{X} + \frac{\Psi^{(1)}_{YYY}}{Y} + \Theta^{(1)}; \quad (27)$$

$$- \frac{P^{(1)}_Y}{Y} = 0; \quad (28)$$

$$\frac{\Psi^{(0)}_Y}{Y} \Theta^{(1)}_X + \frac{\Psi^{(1)}_Y}{Y} \Theta^{(0)}_X - \frac{\Psi^{(0)}_X}{X} \Theta^{(1)}_Y - \frac{\Psi^{(1)}_X}{X} \Theta^{(0)}_Y = \frac{1}{\text{Pr}} \Theta^{(1)}_{YY}. \quad (29)$$

### C. Zeroth-Order Solutions

Equation (25) implies that the zeroth-order pressure gradient exists only in the  $x$ -direction. Then for a vertical plate,  $P_X^{(0)} = -G$ ; i.e., the zeroth-order pressure gradient in the  $x$ -direction is due solely to the weight of the

fluid. Equations (24) and (26), with  $P_X^{(0)} = -G$ , are evidently the equations of the boundary-layer type. The following new variables are introduced:

$$\eta = Y/(5X)^{1/5} \quad (30)$$

$$\Psi^{(0)} = (5X)^{4/5} F(\eta); \quad (31)$$

$$\Theta^{(0)} = (5X)^{1/5} \phi(\eta); \quad (32)$$

where  $\eta$  is a similarity variable, and  $F(\eta)$  and  $\phi(\eta)$  are the functions of  $\eta$  alone. Equations (24) and (26) can be then transformed into the following pair of ordinary simultaneous differential equations:

$$F''' + 4 F F'' - 3 F'^2 + \phi = 0; \quad (33)$$

$$\phi'' + 4 \text{Pr } F \phi' - \text{Pr } F' \phi = 0; \quad (34)$$

where the primes denote differentiations with respect to  $\eta$ . The boundary conditions given in Eq. (13) become

$$F(0) = F'(0) = 0, \quad \phi'(0) = -1, \quad F'(\infty) = \phi(\infty) = 0. \quad (35)$$

But  $\eta = Y(5X)^{-1/5} = (y/x)(\text{Gr}_x^*/5)^{1/5}$ , where  $\text{Gr}_x^*$ , the modified local Grashof number based on  $x$  is equal to  $g\beta x^4 q/k\nu^2$ . Equations (33) and (34), subject to the boundary conditions in Eq. (35), can then be identified as the boundary-layer equations, for a vertical plate with uniform surface heat flux, derived by Sparrow and Gregg.(47)<sup>†</sup> This indicates that the boundary-layer equations developed by these authors can be taken as the zeroth-order approximation. Sparrow and Gregg(47) have solved Eqs. (33) and (34) numerically for Prandtl numbers of 0.1, 1, 10, and 100. Unfortunately, the tables were not available for the present analysis, in which the main interest is for the case of low Prandtl numbers including 0.1. Therefore, numerical calculations were carried out on an IBM 704 to establish the tables of the functions for these Prandtl numbers using the guessed initial conditions for  $F''(0)$  and  $\phi(0)$  given by Sparrow and Gregg.(47) In addition, new solutions for Prandtl numbers of 0.01 and 0.03 were also obtained. Since only three of the five boundary conditions are given at  $\eta = 0$ , guesses had to be made for  $F''(0)$  and  $\phi(0)$  which satisfy  $F'(\infty) = 0$  and  $\phi(\infty) = 0$  after the calculations have been carried out. This iterative process requires extensive time in obtaining the refined solutions. This is especially so for Prandtl numbers less than 0.1 because the Prandtl number has a pronounced effect upon the functions and derivatives, and therefore, a long time is required to perform a single operation of calculation. The major part of the calculations was carried out by AIDE 1 Subroutine obtainable from the SHARE program. The step size of 0.01 for  $\eta$  was used throughout, and no oscillation was observed even with a Prandtl number of 0.01. To be assured of the accuracy

---

<sup>†</sup>In their nomenclature,  $-\theta$  is equal to the present  $\phi$ .

of the results, the solutions thus obtained were compared with the curves of the known solutions of Sparrow and Gregg, and were found to be in good agreement. The accuracy of the first-order perturbation solutions depends entirely upon that of the zeroth-order solutions (as will be seen in Section D below) and the integrated values at large  $\eta$  were found to be quite sensitive to small changes in the guessed initial conditions. Therefore, small increments in the iterative process were necessary for low Prandtl numbers. This high accuracy would not have been necessary if the boundary-layer solutions alone were to be obtained. The functions and derivatives associated with the zeroth-order solutions for various Prandtl numbers are listed, for reference, in Appendix I.

The dimensionless velocity and temperature distributions of the boundary-layer solutions for various Prandtl numbers can be obtained according to the following equations:

$$\frac{x}{\nu} \frac{d}{dx} \left( \frac{Gr^* x}{5} \right)^{2/5} u = F'(\eta); \quad (36)$$

$$\frac{T - T_\infty}{T_w - T_\infty} = \frac{\phi(\eta)}{\phi(0)}; \quad (37)$$

where  $\phi(0)$  is the value at  $\eta = 0$ , and  $T_w$  is the surface temperature of the plate. The dimensionless velocity and temperature distributions thus obtained for Prandtl numbers of 0.01, 0.03, 0.1, 1, 10, and 100 are shown in Figs. 2 and 3, respectively. The dotted lines are for Prandtl numbers of 0.1, 1, 10, and 100, and are the reproduction of Sparrow and Gregg's solutions on an IBM 704 with AIDE 1 program. The solid lines represent two new solutions of the boundary-layer equations for Prandtl numbers of 0.01 and 0.03.

#### D. First-Order Solutions

As Lighthill<sup>(27)</sup> and Kuo<sup>(25)</sup> pointed out, the zeroth-order solution to a nonlinear differential equation may contain a singularity at an isolated point or on a line within the domain of interest. Then the singularity will appear again at the same location in the rest of the successive approximations, and it will become more accentuated as the order of the solution increases. Consequently, the classical singular perturbation method fails to give a valid solution near the singular points; i.e., the solution is not uniformly valid over the entire domain of interest. To overcome this difficulty, Lighthill's technique, usually known

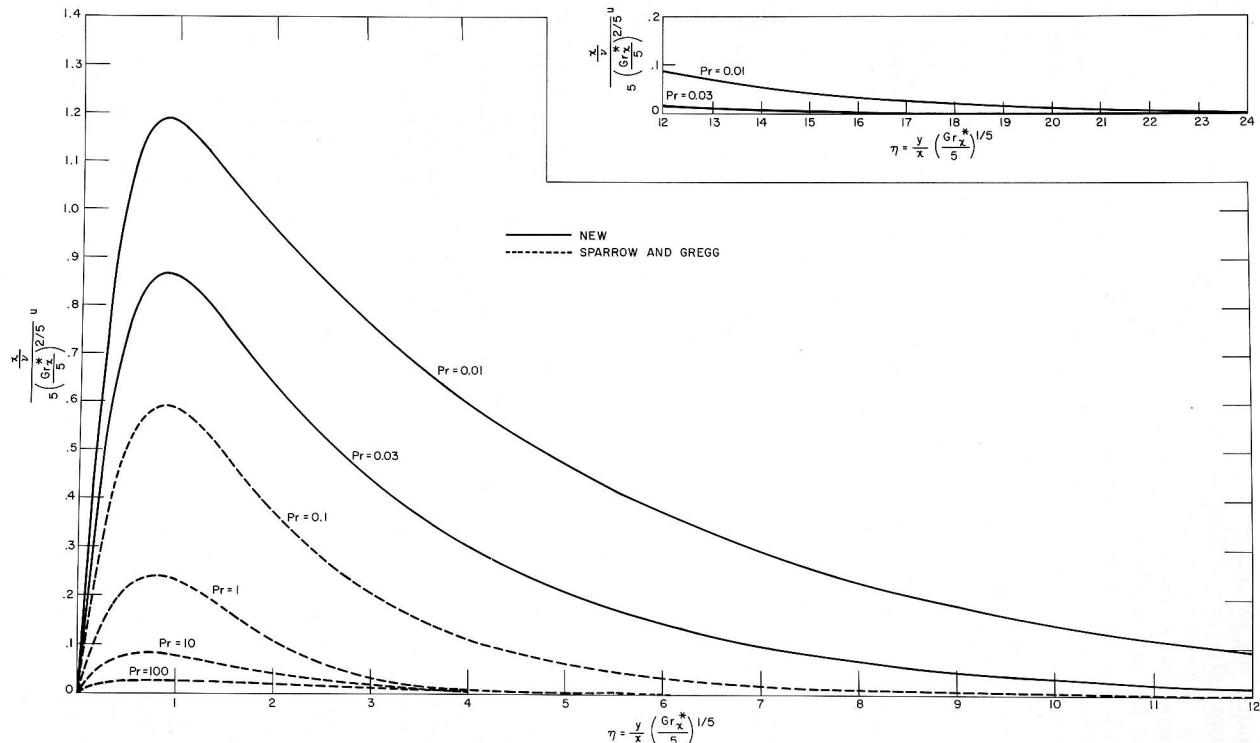


Figure 2. Dimensionless Velocity Distributions for Various Prandtl Numbers

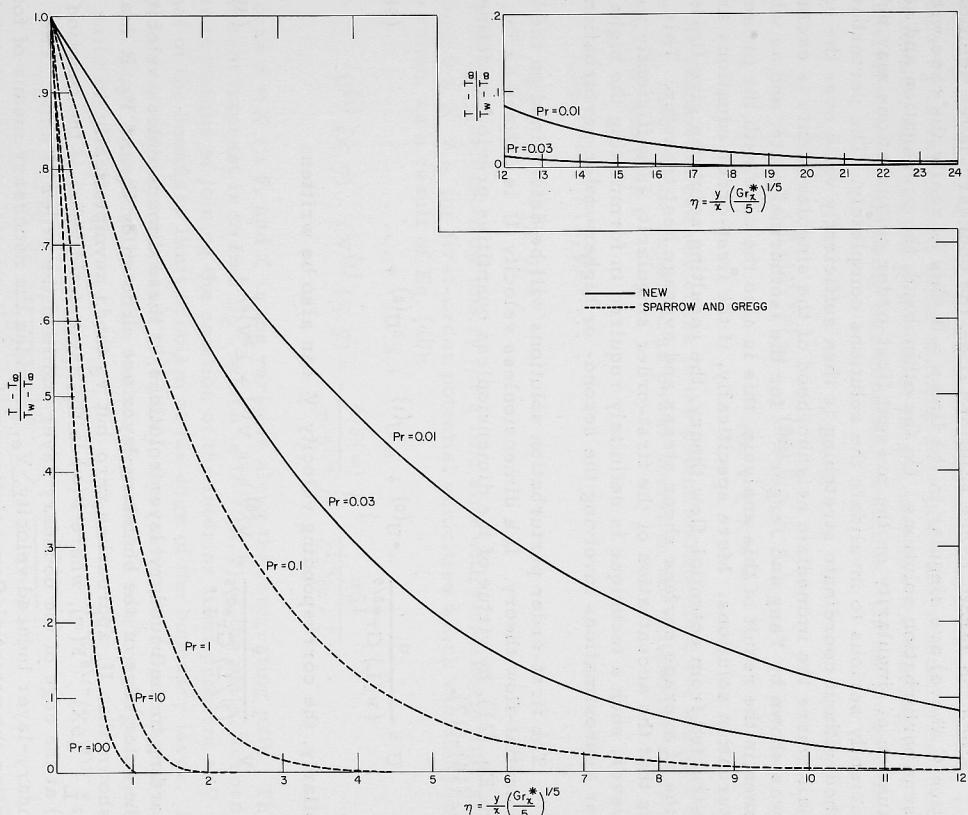


Figure 3. Dimensionless Temperature Distributions for Various Prandtl Numbers

as the PLK method (Poincaré-Lighthill-Kuo method)<sup>(53)</sup> must be applied. In this method, the dependent variables and the independent variables are expanded in power series of the constant parameter (coordinate perturbation or coordinate stretching).

Since the zeroth-order solutions (i.e., the boundary-layer solutions in Section C above) are singular in the line  $x = 0$ , this is really the case with the present perturbation solutions. On the other hand, the propagation and accentuation of singularity on the present first-order perturbations may not be sufficiently serious to invalidate the solutions completely. The perturbation method without coordinate stretching is then sufficiently valid over the entire domain where the immediate neighborhood of the singular points is excluded. As was shown by Yang and Jerger<sup>(56)</sup> for the isothermal plate, and as will be seen in the result of this analysis, this is also the case with the present perturbation solutions. More specifically, if the first-order equations are constructed from potential-flow theory, the resulting solutions give finite values of average surface shear stress and average heat transfer. This implies that the accentuation of the first-order singularity is sufficiently weak. However, such a technique is definitely required in formulating the higher-order approximations involving the second- or higher-order perturbations.

The first-order perturbation solutions will be determined on the basis of potential-flow theory. If a dimensionless velocity  $U$  is defined as  $\Psi_Y$ , then from Eq. (21), by virtue of the dimensionless quantities previously defined in Eqs. (14)-(17),

$$U = \frac{u}{(\nu/L) Gr_L^{*2/5}} = U^{(0)} + \epsilon U^{(1)} + \epsilon^2 U^{(2)} + \dots \quad (38)$$

Similarly, the corresponding velocity  $V$  can also be written:

$$V = \frac{v}{(\nu/L) Gr_L^{*2/5}} = V^{(0)} + \epsilon V^{(1)} + \epsilon^2 V^{(2)} + \dots \quad (39)$$

According to the boundary-layer solutions, if these dimensionless velocities at the outer edge of the boundary-layer are denoted by  $U_e$  and  $V_e$ , it may be shown that  $U_e$  approaches zero but  $V_e$  tends asymptotically to  $-4Gr_L^{*-1/5}(5X)^{-1/5}F(\infty)$ , where  $F(\infty)$  is the asymptotic value of the function  $F(\eta)$  at the edge of the boundary-layer and is of the order of unity. This boundary-layer induced velocity,  $V_e$ , provides the necessary means of formulating the potential-flow solutions and makes possible the smooth joining of the solutions to those of the boundary-layer theory. Since  $V_e$  is of the order of  $Gr_L^{*-1/5}$ , the choice of  $Gr_L^{*-1/5}$  as the constant parameter  $\epsilon$  is justified.

For  $X < 0$ ,  $V_e = 0$ . For  $0 \leq X \leq 1$ ,  $V_e$  takes the asymptotic value  $-4\epsilon(5X)^{-1/5}F(\infty)$ . In the wake region above the trailing edge of the plate, no exact boundary condition can be imposed. As long as laminar flow persists in the wake region, however, the wake may be replaced by the flow lines parallel to the plate with a discontinuity in the inclination at the trailing edge. Then  $V_e = 0$  for  $1 < X \leq \infty$ . This assumption approximates the real situation since  $V_e$  is very small at the trailing edge just before undergoing the discontinuity, and the flow behavior at locations below the trailing edge will be little affected by imposing this boundary condition. On the  $X$ -axis, therefore:

$$V_e = \begin{cases} 0, & \text{for } X < 0; \\ -\epsilon 4(5X)^{-1/5}F(\infty), & \text{for } 0 \leq X \leq 1; \\ 0, & \text{for } 1 < X \leq \infty. \end{cases} \quad (40)$$

If the dimensionless velocities in the potential-flow field are denoted by  $U_p$  and  $V_p$ , they can be expanded as power series in the parameter  $\epsilon$  after Eqs. (38) and (39):

$$U_p = \epsilon U^{(1)}(X, \bar{y}) + \dots ; \quad (41)$$

$$V_p = \epsilon V^{(1)}(X, \bar{y}) + \dots . \quad (42)$$

From the potential-flow theory for a line sink with nonuniform sink strength along  $0 \leq X \leq 1$ , the first-order potential velocities which vanish at infinity are then, as a result of Eq. (40):

$$U^{(1)}(X, \bar{y}) - i V^{(1)}(X, \bar{y}) = -\frac{4}{\pi} \frac{F(\infty)}{5^{1/5}} \int_0^1 \frac{d\zeta}{\zeta^{1/5}(z - \zeta)} ; \quad (43)$$

where  $z = X + i \bar{y}$ , and  $\zeta$  is the variable along the integrating path. The present interest is to find  $U_e^{(1)}$ , which is  $U^{(1)}(X, \bar{y})$  at the plate, according to the technique of joining the solution of the interior limit problem at  $\bar{y} = 0$  to that of the boundary limit problem at the edge of the boundary layer. It is therefore sufficient to obtain  $U^{(1)}(X, 0)$ , the real part of the integral with  $z = X$  (Ref. Appendix II). This is then expressible in a convergent series of  $X$  valid for  $0 < X < 1$ :

$$U_e^{(1)} = U^{(1)}(X, 0) = \frac{20}{\pi} \frac{F(\infty)}{(5X)^{1/5}} \left[ S - \sum_{m=0}^{\infty} \frac{X^{m+1/5}}{5m+1} \right]; \quad (44)$$

where

$$S = \sum_{m=0}^{\infty} \frac{1}{(5m+1)} - \sum_{m=0}^{\infty} \frac{1}{5m+4} = \sum_{m=0}^{\infty} \frac{3}{(5m+1)(5m+4)}$$

$$= 0.864806 \dots \quad (45)$$

Note that  $V_e^{(1)}$  is singular at the leading edge but is finite with discontinuity at the trailing edge, whereas  $U_e^{(1)}$  is singular at both edges.

Now the functions  $P^{(1)}$ ,  $\Psi^{(1)}$ , and  $\Theta^{(1)}$  will be determined. Equation (28) shows that there is no first-order pressure gradient in the  $Y$ -direction. From this, together with Bernoulli's equation, the first-order pressure is now:

$$P^{(1)} = -\frac{1}{2} \epsilon (U_e^{(1)})^2. \quad (46)$$

This shows that the first-order pressure is one order higher, with respect to the parameter  $\epsilon$ , than the corresponding velocity field. Therefore,  $P^{(1)}$  can be deleted from the first-order perturbation equations. This situation differs from that of a corresponding forced flow problem, in which the first-order pressure is of the order of  $U_e^{(1)}$  with respect to  $\epsilon$ . From the power series expansion of  $\Psi$  in Eq. (21), and from the similar expansion of  $U$  in Eq. (38) which is a direct result of the definition  $U = \Psi_Y$ , it is clear that  $U^{(1)} = \Psi_Y^{(1)}$ , and at the edge of the boundary layer  $U^{(1)} = U_e^{(1)}$ . Therefore, let  $\Psi^{(1)}$  and  $\Theta^{(1)}$  be expanded as follows:

$$\Psi^{(1)} = \frac{20 F(\infty)}{\pi} \left[ S f_{00}(\eta) - \sum_{m=0}^{\infty} \frac{x^{m+1/5}}{5m+1} f_m(\eta) \right], \quad (47)$$

$$\Theta^{(1)} = \frac{20 F(\infty)}{\pi} (5x)^{-3/5} \left[ S \theta_{00}(\eta) - \sum_{m=0}^{\infty} \frac{x^{m+1/5}}{5m+1} \theta_m(\eta) \right], \quad (48)$$

where  $f_{00}$ ,  $f_m$ ,  $\theta_{00}$ , and  $\theta_m$  are functions of  $\eta$  alone, and  $\eta$  is defined by Eq. (30). When Eqs. (47) and (48) are substituted in the first-order perturbation equations, namely, Eqs. (27) and (29), the following sets of ordinary differential equations are obtained after some differentiations and manipulations:

$$\left. \begin{aligned} f'''_{00} + 4F f''_{00} - 2F' f'_{00} + \theta_{00} &= 0, \\ \frac{1}{Pr} \theta''_{00} + 4F \theta'_{00} + 3F' \theta_{00} - \phi f'_{00} &= 0; \end{aligned} \right\} \quad (49)$$

$$\left. \begin{aligned} f'''_m + 4 F f''_m - (5m+3) F' f'_m + (5m+1) F'' f_m + \theta_m = 0, \\ \frac{1}{Pr} \theta''_m + 4 F \theta'_m - (5m-2) F' \theta_m - \phi f'_m + (5m+1) \phi' f_m = 0; \end{aligned} \right\} \quad (50)$$

where  $m = 0, 1, 2, \dots$ , and the primes denote the differentiations with respect to  $\eta$ . These equations are to be solved subject to the following boundary conditions:

$$\left. \begin{aligned} f_{00}(0) = f'_{00}(0) = \theta'_{00}(0) = 0, \\ f'_{00}(\infty) = 1, \quad \theta_{00}(\infty) = 0; \end{aligned} \right\} \quad (51)$$

and

$$\left. \begin{aligned} f_m(0) = f'_m(0) = \theta'_m(0) = 0, \\ f'_m(\infty) = 1, \quad \theta_m(\infty) = 0. \end{aligned} \right\} \quad (52)$$

The solutions to Eqs. (49) and (50) have been obtained numerically on an IBM 704 for Prandtl numbers 0.1 and 0.03. These Prandtl numbers were chosen because the results for 0.1 could easily be compared with the boundary-layer solution obtained by Sparrow and Gregg, (47) and the Prandtl number 0.03 is the approximate value for mercury, a typical liquid metal. As with the zeroth-order solutions, since only three of the five boundary conditions are given at  $\eta = 0$ , the guesses had to be made for  $f''_{00}(0)$  and  $\theta_{00}(0)$  in Eqs. (49), and for  $f''_m(0)$  and  $\theta_m(0)$  in Eqs. (50). Equations (49) and (50) are strongly dependent on the solutions of the zeroth-order equations since the zeroth-order functions appear as coefficients in these equations. This required the extremely refined solutions of the zeroth-order equations. These solutions were then regenerated within the computer for a given  $\eta$  and fed into the program designed to solve Eqs. (49) and (50). The values of  $F''(0)$ ,  $\phi(0)$ ,  $F(\infty)$ ,  $f''_{00}(0)$ ,  $\theta_{00}(0)$ ,  $f''_m(0)$ , and  $\theta_m(0)$  associated with the solutions of Eqs. (49) and (50) for Prandtl numbers of 0.1 and 0.03, subject to the boundary conditions in Eqs. (35), (51), and (52), are listed in Table 1. The values of  $m$  were taken from zero to 10 inclusive. No further calculations were carried out for  $m$  larger than 10 since sufficiently accurate results were determined by the first 11 terms in the series. (Additional terms would be required for calculation of temperature profiles at  $X \approx 0.95$  because of slow convergence.) The behavior of the functions  $f'_{00}(\eta)$  obtained by solving Eq. (49) for the two different Prandtl numbers is shown in Fig. 4. The behavior of the functions  $\theta_{00}(\eta)$  for the two corresponding Prandtl numbers is shown in Fig. 5. The functions  $f'_{00}(\eta)$  and  $\theta_{00}(\eta)$  contribute to the first terms in the first-order velocity and temperature perturbations, respectively. The functions  $f'_m(\eta)$  and  $\theta_m(\eta)$  contributing subsequent 11 terms in the first-order perturbations for a Prandtl number of 0.1 are plotted in Figs. 6 and 7, respectively. Figures 8 and 9 describe the behavior of the same functions  $f'_m(\eta)$  and  $\theta_m(\eta)$  for a Prandtl number of 0.03.

Table 1

VALUES OF VARIOUS FUNCTIONS AND DERIVATIVES  
FOR THE FIRST-ORDER PERTURBATIONS

| Pr                   | 0.1                       | 0.03                      |
|----------------------|---------------------------|---------------------------|
| $F''(0)$             | 1.64342                   | 2.46499                   |
| $\phi(0)$            | 2.75070                   | 4.19766                   |
| $F(\infty)$          | 1.53039                   | 3.11498                   |
| $f_{00}''(0)$        | -0.526807                 | -0.773575                 |
| $\theta_{00}(0)$     | -1.06848                  | -1.38641                  |
| $f_m''(0), m = 0$    | $-3.86577 \times 10^{-1}$ | $-5.46359 \times 10^{-1}$ |
| $m = 1$              | $-1.18367 \times 10^{-1}$ | $-1.47089 \times 10^{-1}$ |
| $m = 2$              | $-4.86292 \times 10^{-2}$ | $-5.70665 \times 10^{-2}$ |
| $m = 3$              | $-2.32665 \times 10^{-2}$ | $-2.67592 \times 10^{-2}$ |
| $m = 4$              | $-1.22866 \times 10^{-2}$ | $-1.40921 \times 10^{-2}$ |
| $m = 5$              | $-6.96083 \times 10^{-3}$ | $-8.02305 \times 10^{-3}$ |
| $m = 6$              | $-4.15761 \times 10^{-3}$ | $-4.83118 \times 10^{-3}$ |
| $m = 7$              | $-2.58795 \times 10^{-3}$ | $-3.03542 \times 10^{-3}$ |
| $m = 8$              | $-1.66527 \times 10^{-3}$ | $-1.97215 \times 10^{-3}$ |
| $m = 9$              | $-1.10123 \times 10^{-3}$ | $-1.31675 \times 10^{-3}$ |
| $m = 10$             | $-7.45137 \times 10^{-4}$ | $-8.99357 \times 10^{-4}$ |
| $\theta_m(0), m = 0$ | $-7.73921 \times 10^{-1}$ | -1.00429                  |
| $m = 1$              | $-2.42240 \times 10^{-1}$ | $-3.18972 \times 10^{-1}$ |
| $m = 2$              | $-1.06975 \times 10^{-1}$ | $-1.42903 \times 10^{-1}$ |
| $m = 3$              | $-5.52578 \times 10^{-2}$ | $-7.46617 \times 10^{-2}$ |
| $m = 4$              | $-3.12900 \times 10^{-2}$ | $-4.26954 \times 10^{-2}$ |
| $m = 5$              | $-1.88430 \times 10^{-2}$ | $-2.59479 \times 10^{-2}$ |
| $m = 6$              | $-1.18675 \times 10^{-2}$ | $-1.64876 \times 10^{-2}$ |
| $m = 7$              | $-7.73663 \times 10^{-3}$ | $-1.08425 \times 10^{-2}$ |
| $m = 8$              | $-5.18498 \times 10^{-3}$ | $-7.32919 \times 10^{-3}$ |
| $m = 9$              | $-3.55503 \times 10^{-3}$ | $-5.06800 \times 10^{-3}$ |
| $m = 10$             | $-2.48479 \times 10^{-3}$ | $-3.57200 \times 10^{-3}$ |

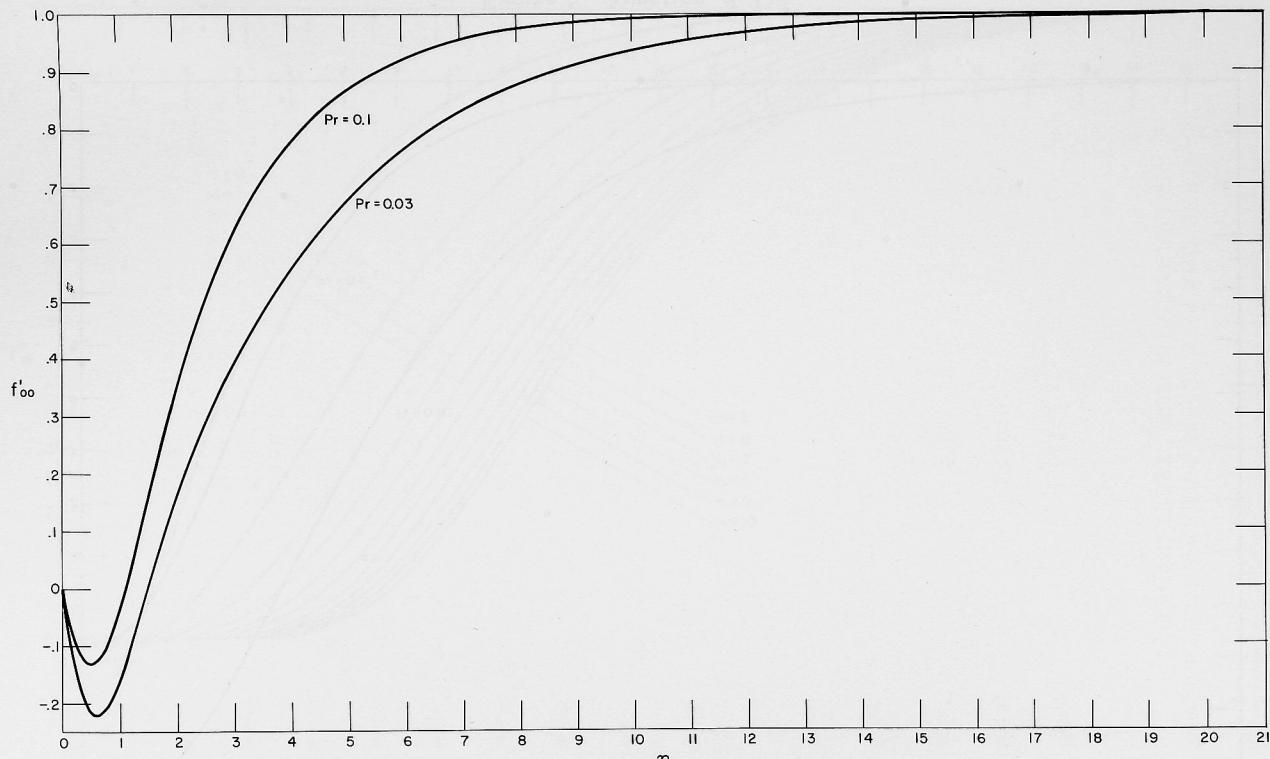


Figure 4. Function  $f'_{oo}(\eta)$

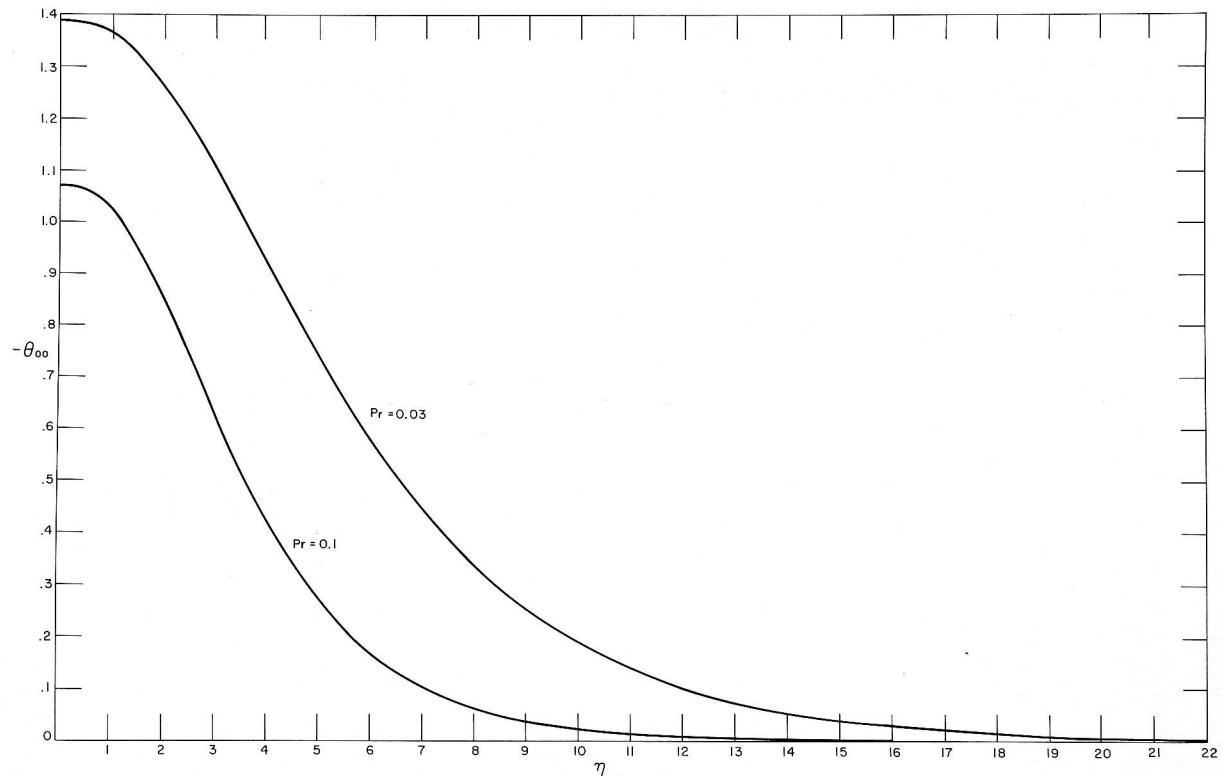


Figure 5. Function  $\theta_{00}(\eta)$

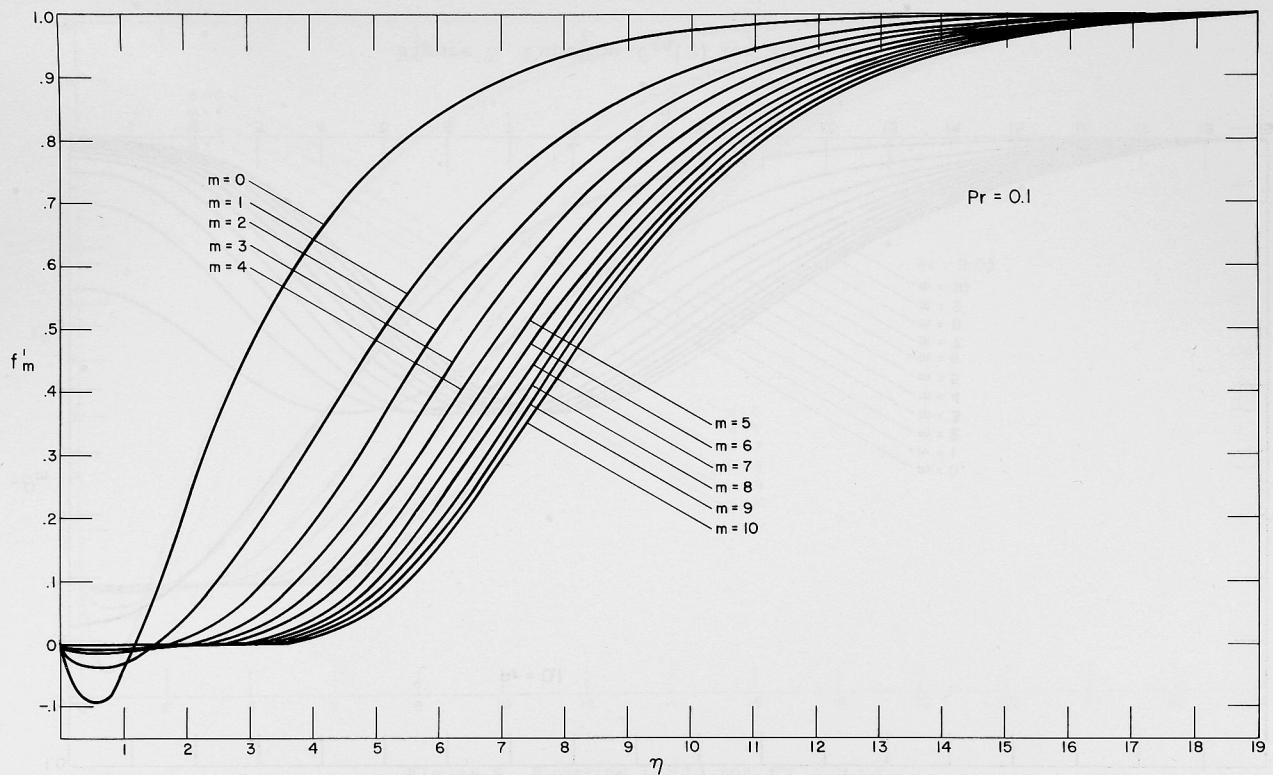


Figure 6. Function  $f'_m(\eta)$  for  $Pr = 0.1$

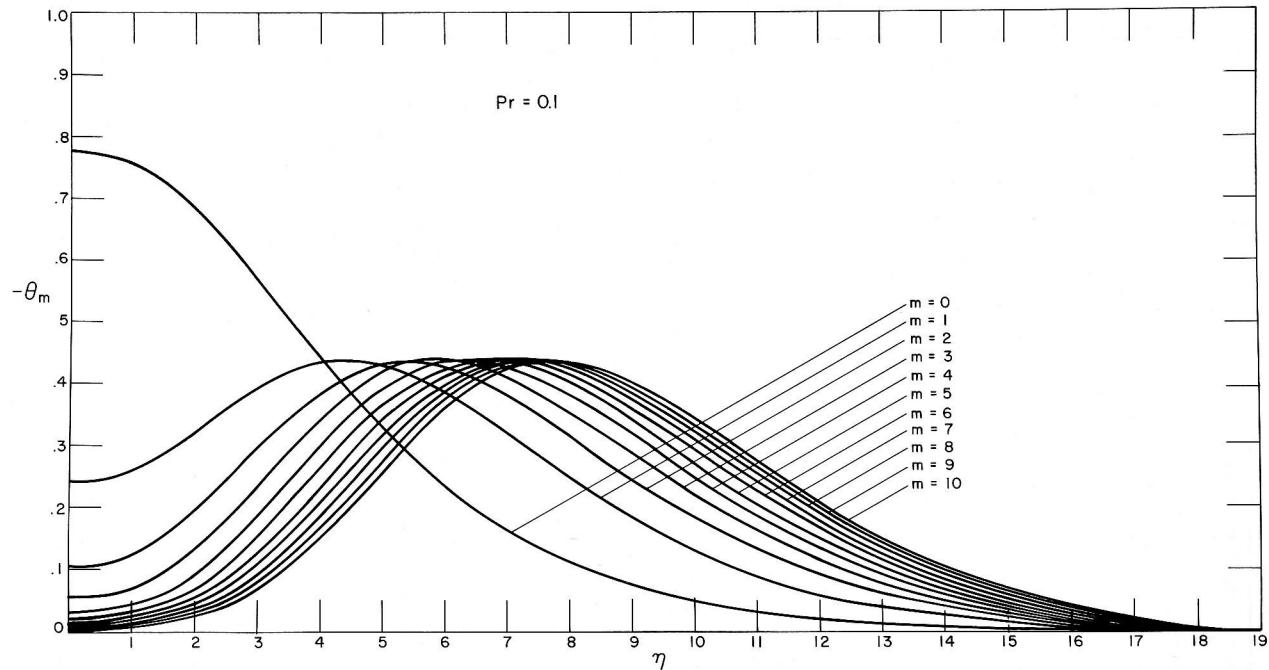


Figure 7. Function  $\theta_m(\eta)$  for  $Pr = 0.1$

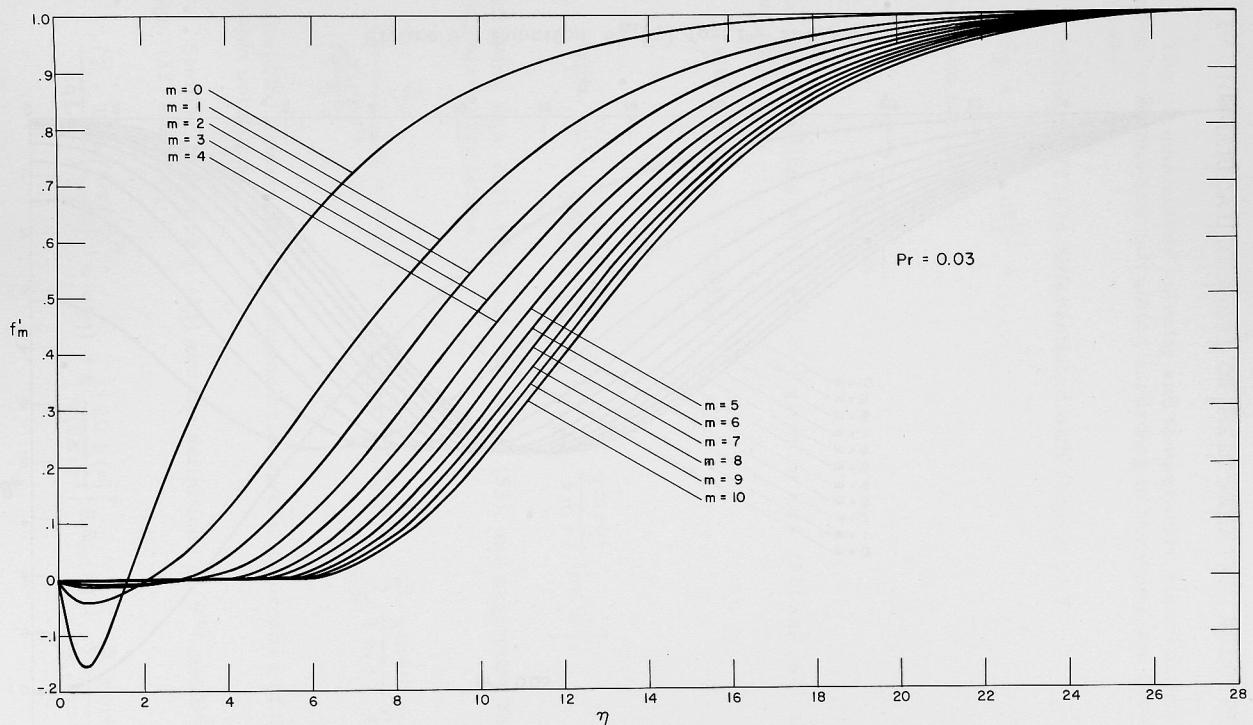


Figure 8. Function  $f'_m(\eta)$  for  $Pr = 0.03$

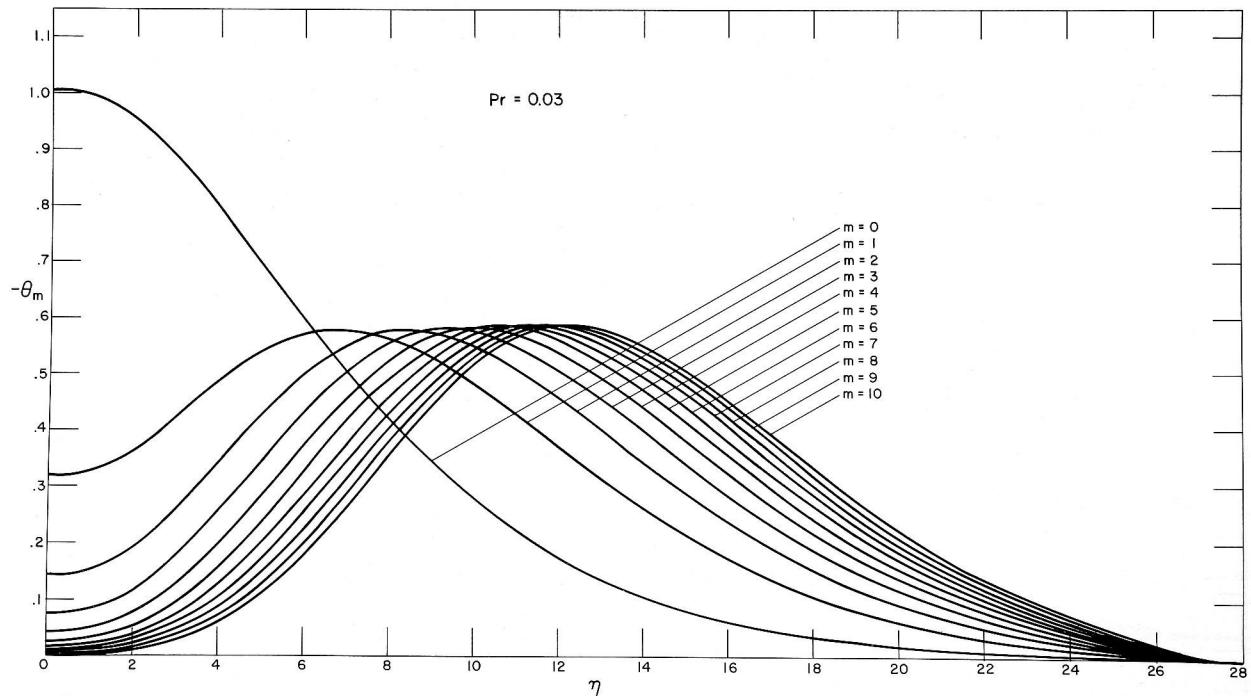


Figure 9. Function  $\theta_m(\eta)$  for  $Pr = 0.03$

#### IV. NUMERICAL RESULTS OF PERTURBATION SOLUTIONS

The dimensionless velocity and temperature distributions of perturbation solutions are calculated according to the equations developed as follows.

When Eq. (21) is differentiated with respect to  $Y$ ,

$$\Psi_Y = \Psi_Y^{(0)} + \epsilon \Psi_Y^{(1)} + \dots \quad (53)$$

Then, from Eq. (38),

$$\Psi_Y = U = \frac{u}{\frac{\nu}{L} Gr_L^{2/5}}. \quad (38)$$

From the definitions of  $\Psi_Y^{(0)}$  and  $\Psi_Y^{(1)}$  given in Eqs. (31) and (47), it may be shown that

$$\Psi_Y^{(0)} = U^{(0)} = F'(\eta)(5X)^{3/5}, \quad (54)$$

and

$$\Psi_Y^{(1)} = U^{(1)} = \frac{20}{\pi} \frac{F(\infty)}{(5X)^{1/5}} \left[ S f_{00}'(\eta) - \sum_{m=0}^{\infty} \frac{X^{m+1/5}}{5m+1} f_m'(\eta) \right]. \quad (55)$$

Substituting Eqs. (38), (54), and (55) in Eq. (53), and neglecting the second- and higher-order terms,

$$\frac{\frac{x}{\nu}}{5 \left( \frac{Gr_L^{2/5}}{5} \right)} u = F'(\eta) + \epsilon \frac{20}{\pi} \frac{F(\infty)}{(5X)^{4/5}} \left[ S f_{00}'(\eta) - \sum_{m=0}^{\infty} \frac{X^{m+1/5}}{5m+1} f_m'(\eta) \right]. \quad (56)$$

Equation (56) determines the dimensionless velocity profiles of the perturbation solutions.

Similarly, from Eq. (23) and the definitions of  $\Theta^{(0)}$  and  $\Theta^{(1)}$  given in Eqs. (32) and (48),

$$\frac{(T - T_\infty)k}{qL} \left( \frac{Gr_L^{2/5}}{5X} \right)^{1/5} = \phi(\eta) + \epsilon \frac{20}{\pi} \frac{F(\infty)}{(5X)^{4/5}} \left[ S \theta_{00}(\eta) - \sum_{m=0}^{\infty} \frac{X^{m+1/5}}{5m+1} \theta_m(\eta) \right]. \quad (57)$$

Equation (57), normalized by dividing through by the value at the wall, is

$$\frac{T - T_{\infty}}{T_w - T_{\infty}} = \frac{\phi(\eta) + \epsilon \frac{20 F(\infty)}{\pi(5X)^{4/5}} \left[ S \theta_{00}(\eta) - \sum_{m=0}^{\infty} \frac{X^{m+1/5}}{5 m + 1} \theta_m(\eta) \right]}{\phi(0) + \epsilon \frac{20 F(\infty)}{\pi(5X)^{4/5}} \left[ S \theta_{00}(0) - \sum_{m=0}^{\infty} \frac{X^{m+1/5}}{5 m + 1} \theta_m(0) \right]}, \quad (58)$$

Equation (58) describes the dimensionless temperature profiles of the perturbation solutions.

The dimensionless velocity and temperature profiles were calculated numerically for Prandtl numbers of 0.1 and 0.03 at different distances from the leading edge of the plate with various values of  $Gr_L^*$  (the modified Grashof number based on  $L$ ) commonly encountered in laminar free convection from a vertical plate having uniform surface heat flux. The results are shown in Figs. 10, 11, 12, 13, and 14 for a Prandtl number of 0.1, and in Figs. 15, 16, 17, 18, and 19 for a Prandtl number of 0.03. For comparison, the boundary-layer solutions represented by Eqs. (36) and (37) are also included as dotted lines in the figures.

For a Prandtl number of 0.1, Fig. 10 shows the effect of the relative distance from the leading edge of the plate on the dimensionless velocity profile when  $Gr_L^*$  is  $10^7$ . For small values of  $\eta$ , the velocity profiles of the perturbation solutions are slightly lower than the corresponding velocity profiles of the boundary-layer solution. These differences increase slightly as the maximum velocities are approached and then diminish until the boundary-layer profile is crossed. Beyond the intersection, the differences become significantly larger as  $\eta$  increases, the magnitude of the differences depending on the relative position  $X$ . Here the positions taken are  $X = \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ , and  $\frac{3}{4}$ . Figure 11 shows the behavior of dimensionless temperature profiles for a Prandtl number of 0.1 at various locations when  $Gr_L^*$  is  $10^7$ . No essential change is seen in the temperature profiles, other than slight lowering of the profiles from the boundary-layer solution for the lower range of  $\eta$ . In particular, at the positions above  $X = \frac{1}{2}$ , the temperature profile can adequately be described by that of the boundary-layer solution. Figure 12 shows the velocity profiles when  $Gr_L^*$  is  $10^9$ . A comparison of Fig. 12 with Fig. 10 shows that the larger the modified Grashof number, the closer are the profiles to the boundary-layer solution. This is more clearly seen in the behavior of corresponding temperature profiles in Fig. 13 where only the profile for  $X = \frac{1}{8}$  is drawn along that of the boundary-layer solution. The effect of  $Gr_L^*$  on the behavior of the velocity profile at the level of the midpoint of the plate length,  $X = \frac{1}{2}$ , is shown in Fig. 14, where the values of  $Gr_L^*$  are  $10^5, 10^6, 10^7, 10^8$ , and  $10^9$ . The corresponding values of the parameter,  $\epsilon$ , are 0.1, 0.063096, 0.039811, 0.025119, and 0.015849, respectively. For a Prandtl number of 0.1, Fig. 14 shows that at  $X = \frac{1}{2}$ , the velocity profile approaches that of the

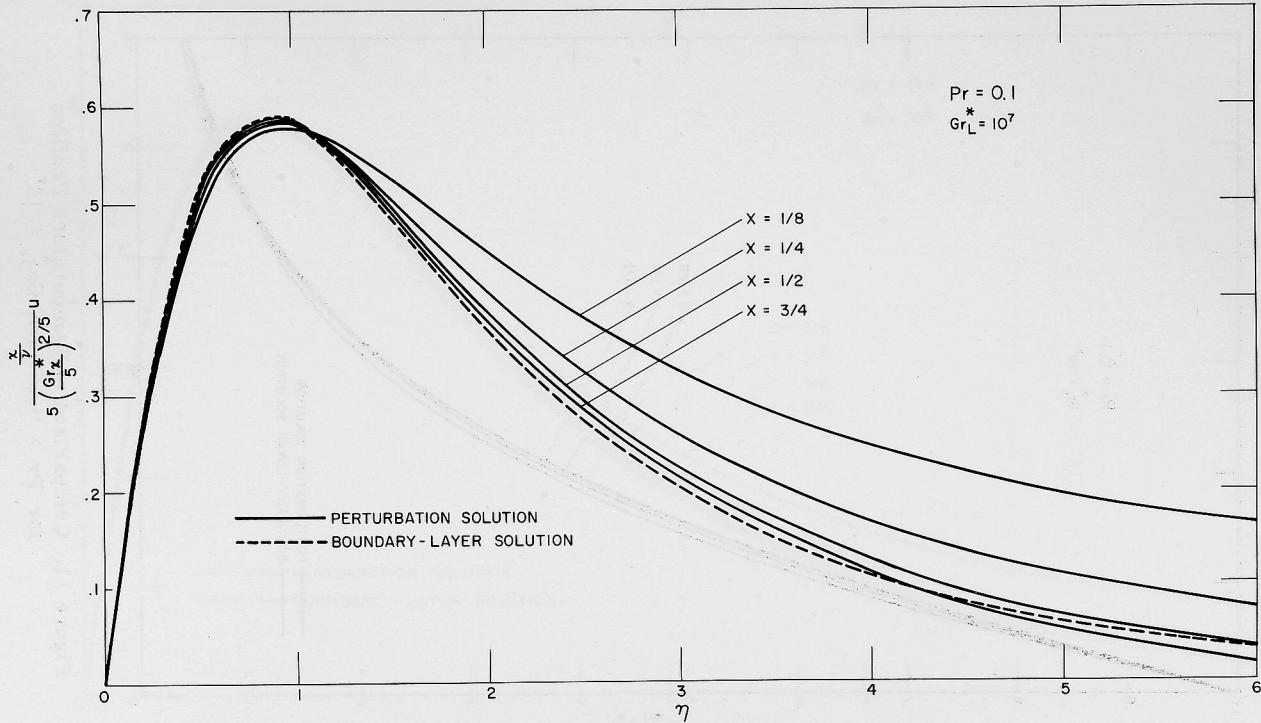


Figure 10. Comparison of Velocity Profiles for  $\Pr = 0.1$  When  $Gr_L^* = 10^7$

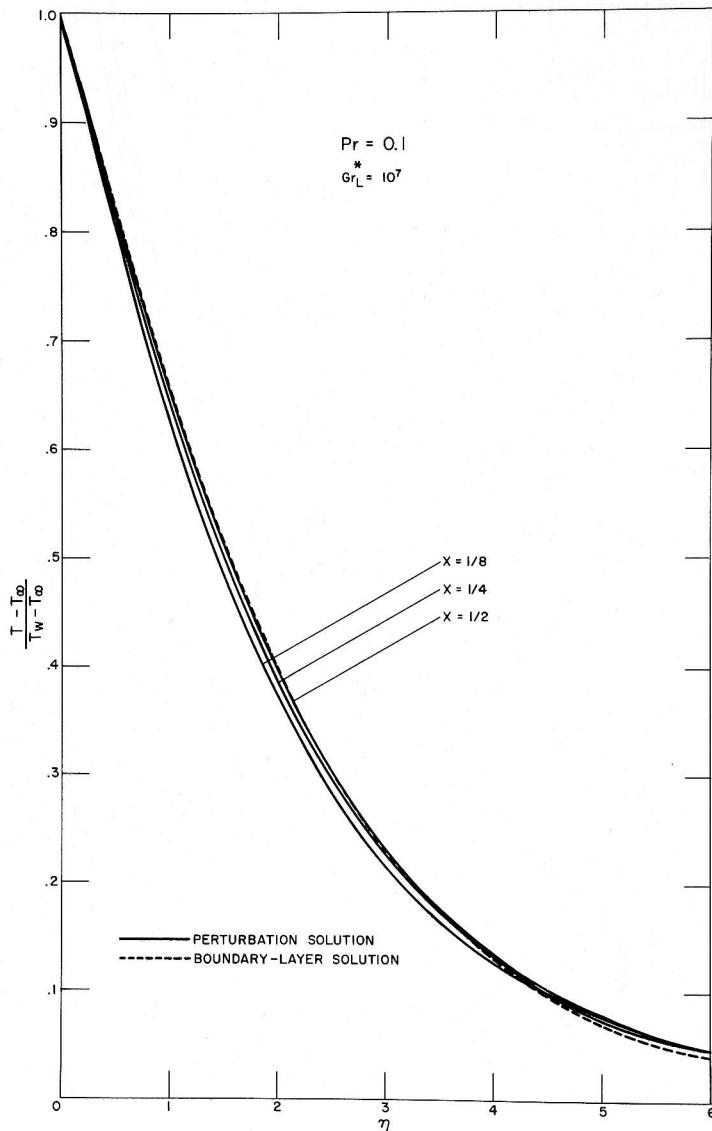


Figure 11. Comparison of Temperature Profiles  
 for  $\Pr = 0.1$  When  $\text{Gr}_L^* = 10^7$

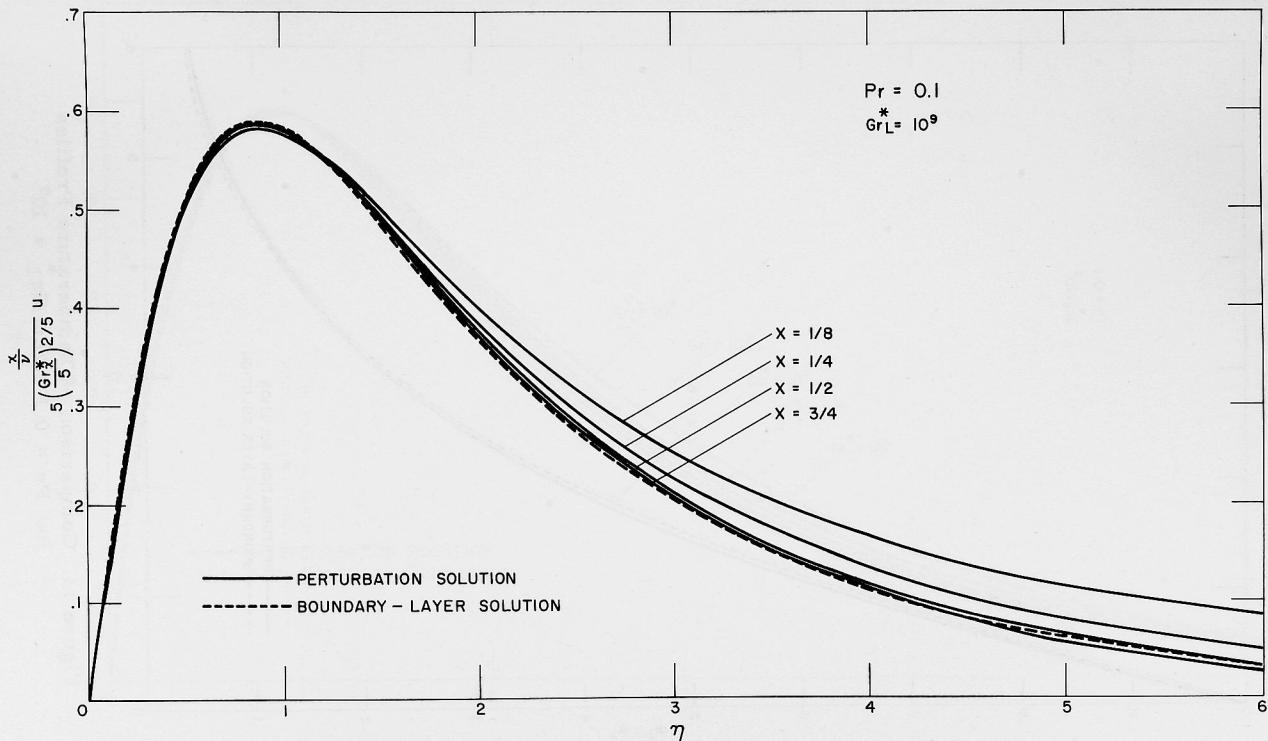


Figure 12. Comparison of Velocity Profiles for  $\Pr = 0.1$  When  $Gr_L^* = 10^9$

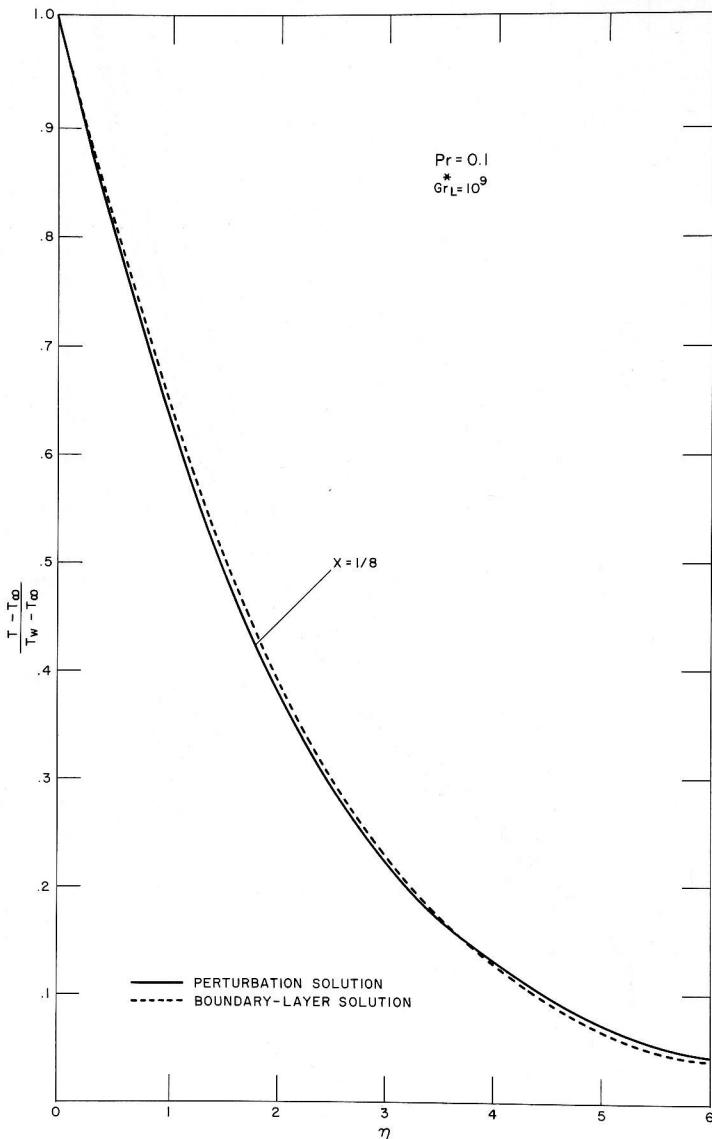


Figure 13. Comparison of Temperature Profiles  
 for  $\Pr = 0.1$  When  $\text{Gr}_L^* = 10^9$

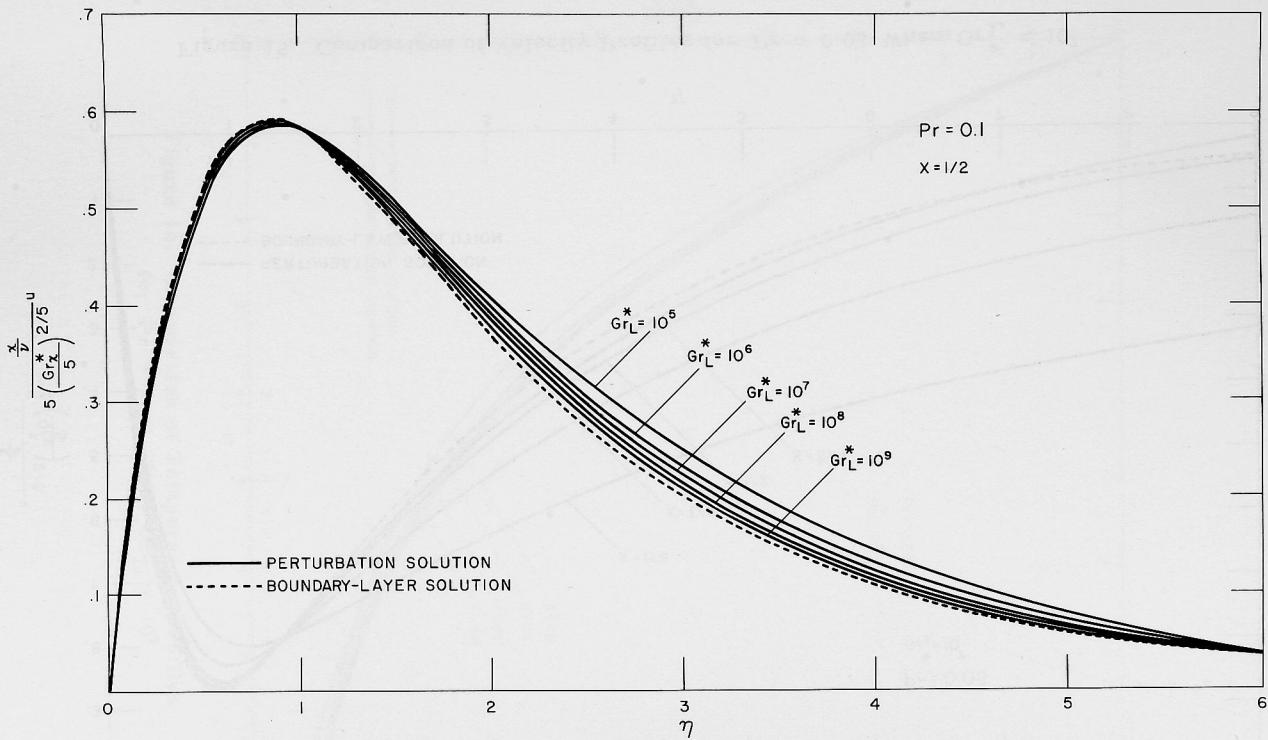


Figure 14. Variation of Velocity Profile with  $Gr_L^*$  at  $X = \frac{1}{2}$  for  $\Pr = 0.1$

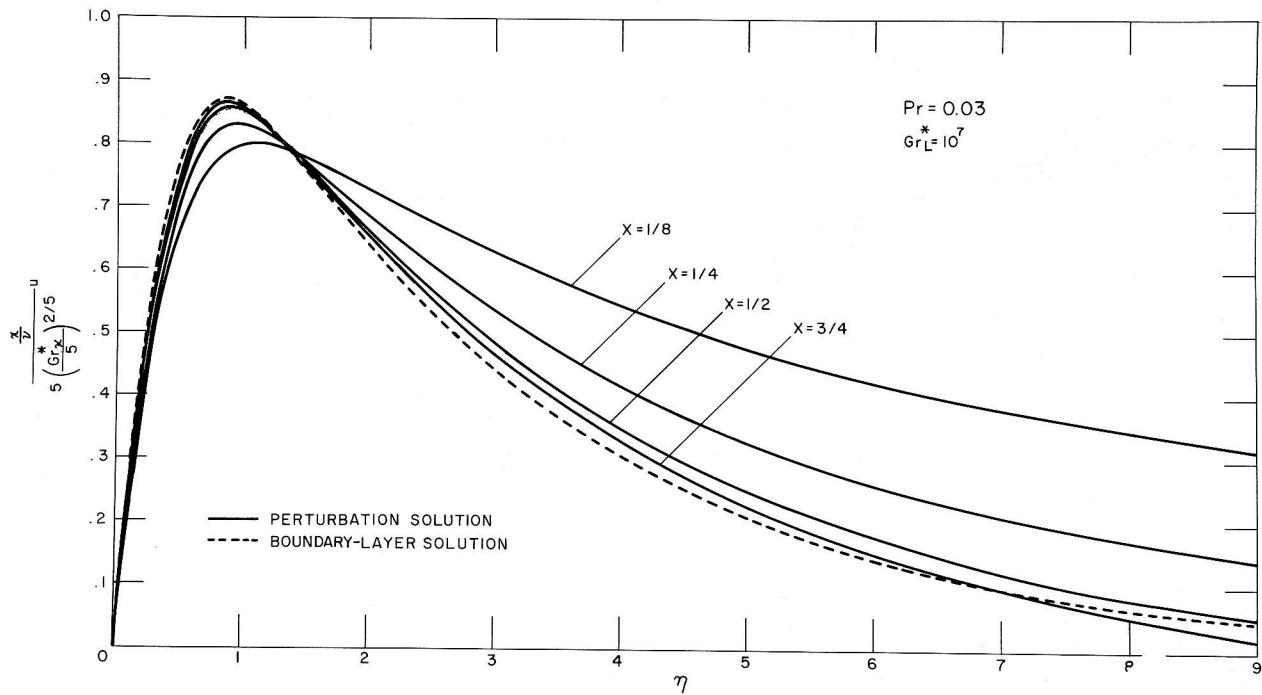


Figure 15. Comparison of Velocity Profiles for  $\text{Pr} = 0.03$  When  $\text{Gr}_L^* = 10^7$

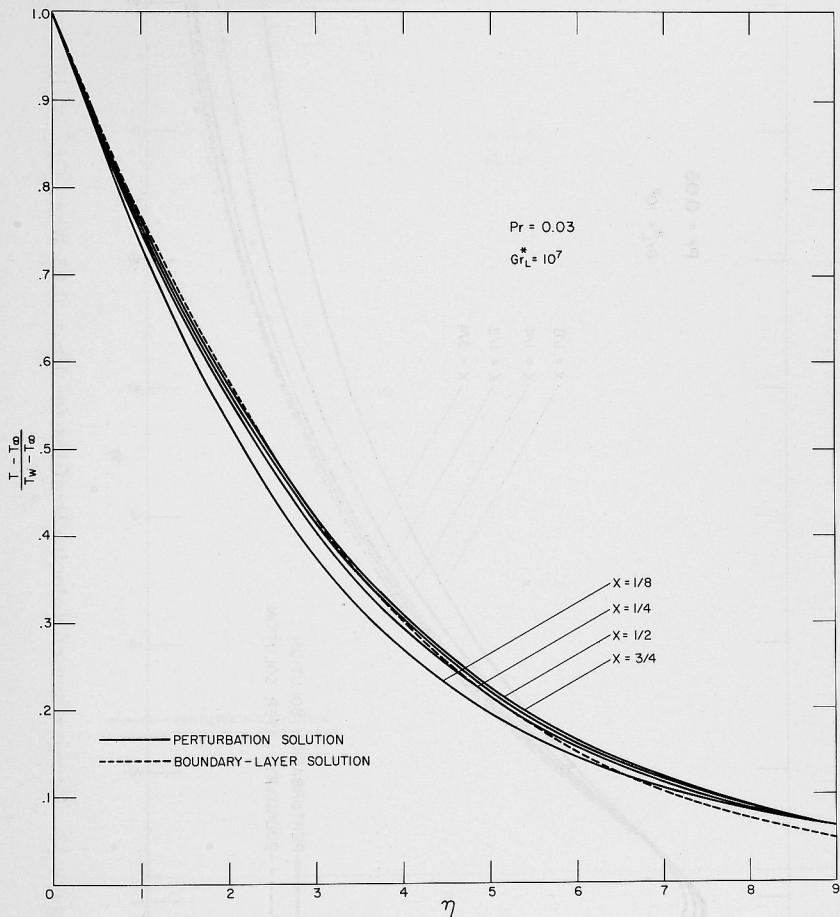


Figure 16. Comparison of Temperature Profiles for  $\text{Pr} = 0.03$  When  $\text{Gr}_L^* = 10^7$

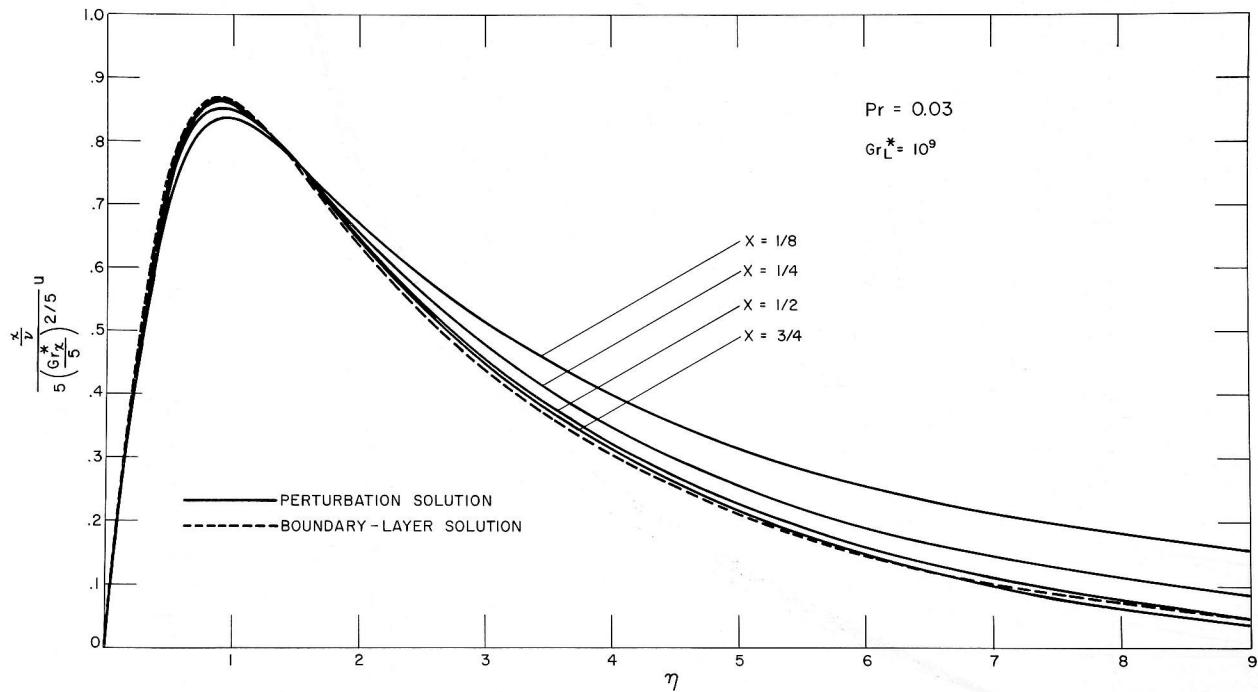


Figure 17. Comparison of Velocity Profiles for  $Pr = 0.03$  When  $Gr_L^* = 10^9$

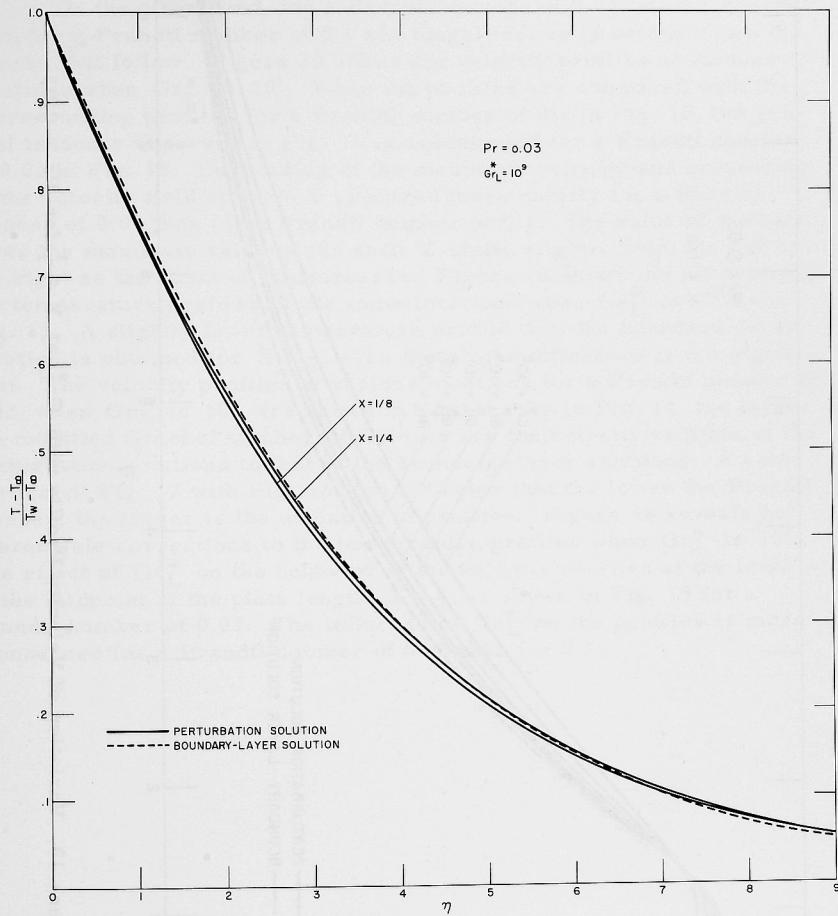


Figure 18. Comparison of Temperature Profiles  
for  $\text{Pr} = 0.03$  When  $\text{Gr}_L^* = 10^9$

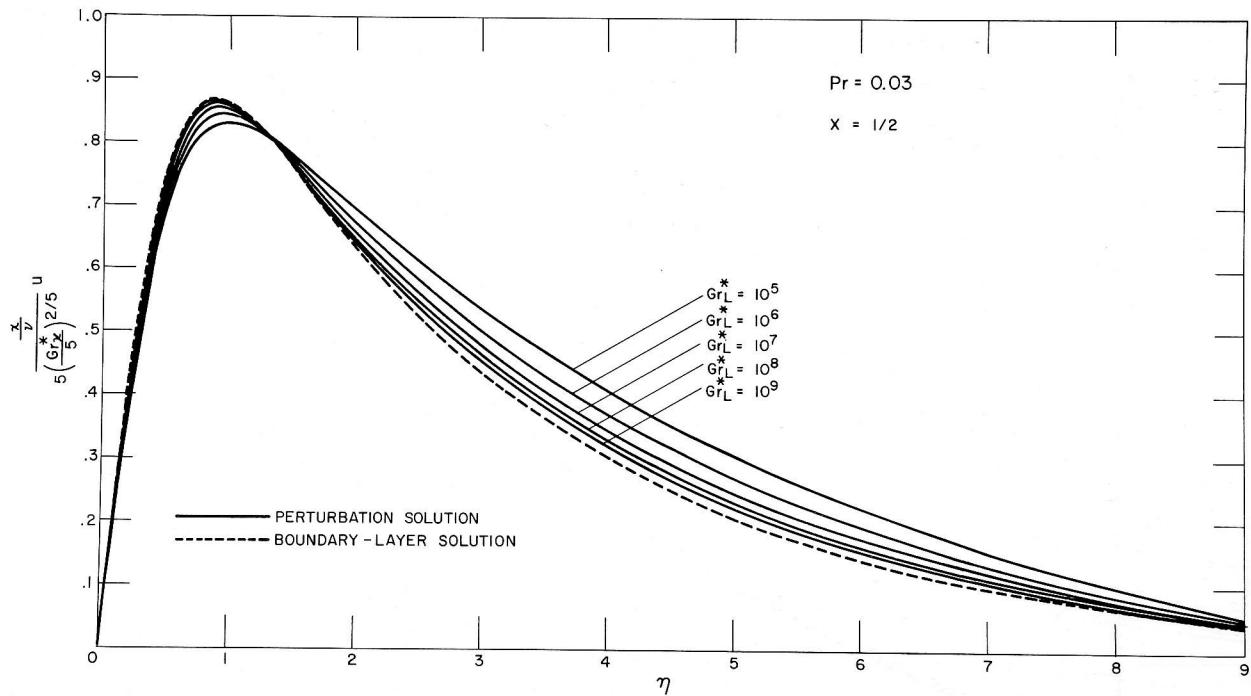


Figure 19. Variation of Velocity Profile with  $Gr_L^*$  at  $X = \frac{1}{2}$  for  $\Pr = 0.03$

boundary-layer solution as  $Gr_L^*$  increases; and at a modified Grashof number of  $10^9$ , the velocity is already similar to that of the boundary-layer solution.

On the other hand, for a Prandtl number of 0.03, all the effects seen for a Prandtl number of 0.1 are magnified, as is evident from the figures that follow. Figure 15 shows the velocity profiles at various locations when  $Gr_L^*$  is  $10^7$ . When the profiles are compared with the corresponding profiles for a Prandtl number of 0.1 in Fig. 10, the general tendency observed in Fig. 10 is accentuated for a Prandtl number of 0.03 in Fig. 15. Decreasing of the maximum velocity and broadening of the velocity field at each  $X$  appeared more clearly for a Prandtl number of 0.03 than for a Prandtl number of 0.1. The value of  $\eta$  which gives the maximum velocity for each  $X$  shifts slightly from the left to the right as the value of  $X$  decreases. Figure 16 shows the behavior of the temperature profiles at the same locations when  $Gr_L^*$  is  $10^7$  as in Fig. 15. A slightly lower temperature profile than the boundary-layer solution is obtained for  $X = \frac{1}{8}$ , even though the difference is not significant. The velocity profiles at various locations for a Prandtl number of 0.03, when  $Gr_L^*$  is  $10^9$ , are shown in Fig. 17. As in Fig. 12, the larger the modified Grashof number, the closer are the velocity profiles of the perturbation solutions to that of the boundary-layer solutions. A comparison of Fig. 17 with Fig. 12 also indicates that the lower the Prandtl number, the larger is the deviation in profiles. Figure 18 reveals no appreciable corrections to the temperature profiles when  $Gr_L^*$  is  $10^9$ . The effect of  $Gr_L^*$  on the behavior of the velocity profiles at the level of the midpoint of the plate length,  $X = \frac{1}{2}$ , is shown in Fig. 19 for a Prandtl number of 0.03. The influence of  $Gr_L^*$  on the profiles is more pronounced for a Prandtl number of 0.03 than for 0.1.

## V. CONCLUSIONS

A first-order perturbation method has been developed for solutions of the problem of two-dimensional free convection in an incompressible viscous fluid at rest from a uniformly heated vertical plate with both leading and trailing edges. By a perturbation method, a series of dimensionless velocity and temperature distributions was obtained for Prandtl numbers of 0.1 and 0.03.

Because the propagation of singularities in the velocity and temperature fields from the zeroth-order to the first-order perturbations is weak, the solutions obtained without using a method of coordinate perturbation are sufficiently valid in a domain where the immediate neighborhood of the leading and trailing edges is excluded. If the second-order perturbation solutions are attempted, an application of the PLK method is definitely required to prevent the perturbation series from diverging.

The first-order potential-flow solution is based on the variable vertical velocity induced by the horizontal velocity on the edge of the boundary-layer. The perturbation solution therefore varies with the value of  $X$  for a given modified Grashof number. As a result, the maximum velocity for a small value of  $X$  is appreciably less than that represented by the boundary-layer solution even though the average velocity for the entire field is increased considerably in the perturbation solution. As  $X$  increases, both the velocity and temperature profiles gradually approach the values given by boundary-layer theory.

On the other hand, the modified Grashof number based on  $L$  contributes to the velocity and temperature distributions, giving different profiles depending on the magnitude of the parameter. As the values of the modified Grashof number increase, the perturbation solutions again approach the boundary-layer solution. The order of magnitude of the modified Grashof number at which the perturbation solutions asymptotically coincide with the boundary-layer solution can be approximately deduced. At  $X = \frac{1}{2}$ , for example, this value is approximately  $10^{10}$  for the Prandtl number of 0.1, and approximately  $10^{11}$  for the Prandtl number of 0.03. However, since the transition from laminar to turbulent flow takes place around this range, the boundary-layer solutions are probably not accurate above this range.

A comparison of the results obtained for two Prandtl numbers shows that for a given modified Grashof number, the lower Prandtl numbers give larger deviations from the boundary-layer solution. Consequently, if the same perturbation calculations are carried out for a Prandtl number of 0.01, the deviations from the boundary-layer solution would be larger than for a Prandtl number of 0.03, especially in the velocity distributions.

## VI. APPENDICES



## APPENDIX I

FUNCTIONS F AND  $\phi$  AND DERIVATIVES FOR  
VARIOUS PRANDTL NUMBERS $Pr = 0.0100$ 

| $\eta$ | F           | F'          | F''           | $\phi$      | $\phi'$      |
|--------|-------------|-------------|---------------|-------------|--------------|
| 0.     | 0.          | 0.          | 3.51956E 00   | 6.30448E 00 | -1.00000E 00 |
| 0.100  | 1.65532E-02 | 3.20700E-01 | 2.89807E 00   | 6.20452E 00 | -9.98945E-01 |
| 0.200  | 6.21142E-02 | 5.80693E-01 | 2.30881E 00   | 6.10475E 00 | -9.95995E-01 |
| 0.300  | 1.30806E-01 | 7.84150E-01 | 1.77001E 00   | 6.00537E 00 | -9.91461E-01 |
| 0.400  | 2.17250E-01 | 9.36829E-01 | 1.29505E 00   | 5.90651E 00 | -9.85630E-01 |
| 0.500  | 3.16706E-01 | 1.04556E 00 | 8.91969E-01   | 5.80828E 00 | -9.78760E-01 |
| 0.600  | 4.25143E-01 | 1.11771E 00 | 5.63233E-01   | 5.71078E 00 | -9.71072E-01 |
| 0.700  | 5.39273E-01 | 1.16060E 00 | 3.06114E-01   | 5.61408E 00 | -9.62746E-01 |
| 0.800  | 6.56518E-01 | 1.18109E 00 | 1.13672E-01   | 5.51825E 00 | -9.53929E-01 |
| 0.900  | 7.74945E-01 | 1.18517E 00 | -2.37870E-02  | 5.42331E 00 | -9.44733E-01 |
| 1.000  | 8.93171E-01 | 1.17780E 00 | -1.17135E-01  | 5.32931E 00 | -9.35241E-01 |
| 1.100  | 1.01025E 00 | 1.16286E 00 | -1.77016E-01  | 5.23627E 00 | -9.25514E-01 |
| 1.200  | 1.12559E 00 | 1.14320E 00 | -2.12849E-01  | 5.14421E 00 | -9.15595E-01 |
| 1.300  | 1.23880E 00 | 1.12083E 00 | -2.32293E-01  | 5.05316E 00 | -9.05516E-01 |
| 1.400  | 1.34971E 00 | 1.09709E 00 | -2.41140E-01  | 4.96312E 00 | -8.95300E-01 |
| 1.500  | 1.45820E 00 | 1.07282E 00 | -2.43490E-01  | 4.87410E 00 | -8.84964E-01 |
| 1.600  | 1.56427E 00 | 1.04852E 00 | -2.42068E-01  | 4.78613E 00 | -8.74523E-01 |
| 1.700  | 1.66792E 00 | 1.02447E 00 | -2.38590E-01  | 4.69920E 00 | -8.63987E-01 |
| 1.800  | 1.76918E 00 | 1.00083E 00 | -2.34075E-01  | 4.61333E 00 | -8.53369E-01 |
| 1.900  | 1.86810E 00 | 9.77673E-01 | -2.29097E-01  | 4.52853E 00 | -8.42678E-01 |
| 2.000  | 1.96473E 00 | 9.55019E-01 | -2.23964E-01  | 4.44480E 00 | -8.31924E-01 |
| 2.100  | 2.05912E 00 | 9.32880E-01 | -2.18831E-01  | 4.36215E 00 | -8.21116E-01 |
| 2.200  | 2.15132E 00 | 9.11250E-01 | -2.13771E-01  | 4.28058E 00 | -8.10262E-01 |
| 2.300  | 2.24139E 00 | 8.90122E-01 | -2.08814E-01  | 4.20009E 00 | -7.99372E-01 |
| 2.400  | 2.32936E 00 | 8.69484E-01 | -2.03974E-01  | 4.12070E 00 | -7.88454E-01 |
| 2.500  | 2.41530E 00 | 8.49323E-01 | -1.99251E-01  | 4.04240E 00 | -7.77516E-01 |
| 2.600  | 2.49924E 00 | 8.29630E-01 | -1.94643E-01  | 3.96520E 00 | -7.66566E-01 |
| 2.700  | 2.58124E 00 | 8.10391E-01 | -1.90149E-01  | 3.88909E 00 | -7.55613E-01 |
| 2.800  | 2.66134E 00 | 7.91596E-01 | -1.85764E-01  | 3.81408E 00 | -7.44662E-01 |
| 2.900  | 2.73957E 00 | 7.73235E-01 | -1.81484E-01  | 3.74016E 00 | -7.33722E-01 |
| 3.000  | 2.81600E 00 | 7.55296E-01 | -1.77307E-01  | 3.66733E 00 | -7.22800E-01 |
| 3.100  | 2.89065E 00 | 7.37770E-01 | -1.73229E-01  | 3.59560E 00 | -7.11901E-01 |
| 3.200  | 2.96357E 00 | 7.20647E-01 | -1.69247E-01  | 3.52495E 00 | -7.01033E-01 |
| 3.300  | 3.03479E 00 | 7.03917E-01 | -1.65359E-01  | 3.45539E 00 | -6.90202E-01 |
| 3.400  | 3.10436E 00 | 6.87572E-01 | -1.61561E-01  | 3.38691E 00 | -6.79741E-01 |
| 3.500  | 3.17232E 00 | 6.71602E-01 | -1.57852E-01  | 3.31951E 00 | -6.68674E-01 |
| 3.600  | 3.23869E 00 | 6.55999E-01 | -1.54228E-01  | 3.25317E 00 | -6.57987E-01 |
| 3.700  | 3.30353E 00 | 6.40754E-01 | -1.50688E-01  | 3.18791E 00 | -6.47359E-01 |
| 3.800  | 3.36686E 00 | 6.25859E-01 | -1.47229E-01  | 3.12370E 00 | -6.36795E-01 |
| 3.900  | 3.42871E 00 | 6.11305E-01 | -1.43849E-01  | 3.06054E 00 | -6.26298E-01 |
| 4.000  | 3.48913E 00 | 5.97086E-01 | -1.40547E-01  | 2.99844E 00 | -6.15875E-01 |
| 4.100  | 3.54814E 00 | 5.83194E-01 | -1.37320E-01  | 2.93737E 00 | -6.05282E-01 |
| 4.200  | 3.60578E 00 | 5.69620E-01 | -1.34167E-01  | 2.87733E 00 | -5.95262E-01 |
| 4.300  | 3.66207E 00 | 5.56358E-01 | -1.31085E-01  | 2.81831E 00 | -5.85086E-01 |
| 4.400  | 3.71706E 00 | 5.43400E-01 | -1.28074E-01  | 2.76031E 00 | -5.74986E-01 |
| 4.500  | 3.77076E 00 | 5.30741E-01 | -1.25130E-01  | 2.70331E 00 | -5.64983E-01 |
| 4.600  | 3.82322E 00 | 5.18372E-01 | -1.222254E-01 | 2.64731E 00 | -5.55075E-01 |
| 4.700  | 3.87445E 00 | 5.06288E-01 | -1.19443E-01  | 2.59229E 00 | -5.45262E-01 |
| 4.800  | 3.92448E 00 | 4.94481E-01 | -1.16696E-01  | 2.53825E 00 | -5.35550E-01 |
| 4.900  | 3.97335E 00 | 4.82947E-01 | -1.14010E-01  | 2.48518E 00 | -5.25939E-01 |
| 5.000  | 4.02108E 00 | 4.71677E-01 | -1.11386E-01  | 2.43306E 00 | -5.16432E-01 |
| 5.100  | 4.06770E 00 | 4.60667E-01 | -1.08821E-01  | 2.38189E 00 | -5.07031E-01 |
| 5.200  | 4.11322E 00 | 4.49911E-01 | -1.06314E-01  | 2.33165E 00 | -4.97739E-01 |
| 5.300  | 4.15769E 00 | 4.39403E-01 | -1.03864E-01  | 2.28234E 00 | -4.88555E-01 |
| 5.400  | 4.20111E 00 | 4.29137E-01 | -1.01464E-01  | 2.23394E 00 | -4.79438E-01 |
| 5.500  | 4.24352E 00 | 4.19107E-01 | -9.91289E-02  | 2.18644E 00 | -4.70524E-01 |
| 5.600  | 4.28494E 00 | 4.09309E-01 | -9.68415E-02  | 2.13983E 00 | -4.61677E-01 |
| 5.700  | 4.32539E 00 | 3.99737E-01 | -9.46059E-02  | 2.09410E 00 | -4.52946E-01 |
| 5.800  | 4.36490E 00 | 3.90386E-01 | -9.24210E-02  | 2.04924E 00 | -4.44330E-01 |
| 5.900  | 4.40348E 00 | 3.81251E-01 | -9.02856E-02  | 2.00523E 00 | -4.35831E-01 |

**Pr = 0.0100 (Contd.)**

| $\eta$ | F           | F'          | F''          | $\phi$      | $\phi'$      |
|--------|-------------|-------------|--------------|-------------|--------------|
| 6.000  | 4.44115E+00 | 3.72327E-01 | -8.81986E-02 | 1.96207E+00 | -4.27448E-01 |
| 6.100  | 4.47795E+00 | 3.63610E-01 | -8.61590E-02 | 1.91974E+00 | -4.19183E-01 |
| 6.200  | 4.51388E+00 | 3.55094E-01 | -8.41657E-02 | 1.87823E+00 | -4.11035E-01 |
| 6.300  | 4.54897E+00 | 3.46775E-01 | -8.22176E-02 | 1.83752E+00 | -4.03067E-01 |
| 6.400  | 4.58324E+00 | 3.38649E-01 | -8.03138E-02 | 1.79762E+00 | -3.95095E-01 |
| 6.500  | 4.61671E+00 | 3.30711E-01 | -7.84533E-02 | 1.75850E+00 | -3.87302E-01 |
| 6.600  | 4.64939E+00 | 3.22957E-01 | -7.66351E-02 | 1.72016E+00 | -3.79627E-01 |
| 6.700  | 4.68131E+00 | 3.15383E-01 | -7.48583E-02 | 1.68257E+00 | -3.72071E-01 |
| 6.800  | 4.71248E+00 | 3.07984E-01 | -7.31219E-02 | 1.64574E+00 | -3.64632E-01 |
| 6.900  | 4.74291E+00 | 3.00757E-01 | -7.14251E-02 | 1.60964E+00 | -3.57311E-01 |
| 7.000  | 4.77263E+00 | 2.93698E-01 | -6.97670E-02 | 1.57427E+00 | -3.50106E-01 |
| 7.100  | 4.80166E+00 | 2.86802E-01 | -6.81467E-02 | 1.53962E+00 | -3.43018E-01 |
| 7.200  | 4.83000E+00 | 2.80067E-01 | -6.65633E-02 | 1.50567E+00 | -3.36046E-01 |
| 7.300  | 4.85767E+00 | 2.73488E-01 | -6.50162E-02 | 1.47240E+00 | -3.29189E-01 |
| 7.400  | 4.88470E+00 | 2.67063E-01 | -6.35044E-02 | 1.43982E+00 | -3.22448E-01 |
| 7.500  | 4.91109E+00 | 2.60786E-01 | -6.20272E-02 | 1.40791E+00 | -3.15820E-01 |
| 7.600  | 4.93686E+00 | 2.54656E-01 | -6.05837E-02 | 1.37666E+00 | -3.09305E-01 |
| 7.700  | 4.96203E+00 | 2.48669E-01 | -5.91733E-02 | 1.34605E+00 | -3.02902E-01 |
| 7.800  | 4.98660E+00 | 2.42820E-01 | -5.77953E-02 | 1.31607E+00 | -2.96611E-01 |
| 7.900  | 5.01060E+00 | 2.37108E-01 | -5.64488E-02 | 1.28672E+00 | -2.90430E-01 |
| 8.000  | 5.03403E+00 | 2.31530E-01 | -5.51332E-02 | 1.25798E+00 | -2.84359E-01 |
| 8.100  | 5.05691E+00 | 2.26081E-01 | -5.38478E-02 | 1.22984E+00 | -2.78396E-01 |
| 8.200  | 5.07925E+00 | 2.20759E-01 | -5.25919E-02 | 1.20230E+00 | -2.72540E-01 |
| 8.300  | 5.10106E+00 | 2.15561E-01 | -5.13649E-02 | 1.17533E+00 | -2.66796E-01 |
| 8.400  | 5.12236E+00 | 2.10465E-01 | -5.01660E-02 | 1.14894E+00 | -2.61145E-01 |
| 8.500  | 5.14316E+00 | 2.05527E-01 | -4.89948E-02 | 1.12310E+00 | -2.55664E-01 |
| 8.600  | 5.16347E+00 | 2.00685E-01 | -4.78505E-02 | 1.09781E+00 | -2.50166E-01 |
| 8.700  | 5.18330E+00 | 1.95956E-01 | -4.67326E-02 | 1.07306E+00 | -2.44830E-01 |
| 8.800  | 5.20267E+00 | 1.91333E-01 | -4.56404E-02 | 1.04884E+00 | -2.39593E-01 |
| 8.900  | 5.22158E+00 | 1.86827E-01 | -4.45735E-02 | 1.02514E+00 | -2.34456E-01 |
| 9.000  | 5.24004E+00 | 1.82422E-01 | -4.35311E-02 | 1.00195E+00 | -2.29416E-01 |
| 9.100  | 5.25806E+00 | 1.78120E-01 | -4.25128E-02 | 9.79256E+00 | -2.24472E-01 |
| 9.200  | 5.27566E+00 | 1.73919E-01 | -4.15180E-02 | 9.57052E+00 | -2.19624E-01 |
| 9.300  | 5.29285E+00 | 1.69816E-01 | -4.05462E-02 | 9.35328E+00 | -2.14870E-01 |
| 9.400  | 5.30963E+00 | 1.65809E-01 | -3.95969E-02 | 9.14075E+00 | -2.10208E-01 |
| 9.500  | 5.32620E+00 | 1.61896E-01 | -3.86695E-02 | 8.93283E+00 | -2.05637E-01 |
| 9.600  | 5.34201E+00 | 1.58074E-01 | -3.77636E-02 | 8.72994E+00 | -2.01156E-01 |
| 9.700  | 5.35763E+00 | 1.54343E-01 | -3.68787E-02 | 8.53049E+00 | -1.96764E-01 |
| 9.800  | 5.37289E+00 | 1.50698E-01 | -3.60143E-02 | 8.33589E+00 | -1.92459E-01 |
| 9.900  | 5.38778E+00 | 1.47139E-01 | -3.51699E-02 | 8.14554E+00 | -1.88240E-01 |
| 10.000 | 5.40232E+00 | 1.43663E-01 | -3.43451E-02 | 7.95938E+00 | -1.84105E-01 |
| 10.100 | 5.41651E+00 | 1.40269E-01 | -3.35394E-02 | 7.77731E+00 | -1.80054E-01 |
| 10.200 | 5.43037E+00 | 1.36955E-01 | -3.27525E-02 | 7.59924E+00 | -1.76085E-01 |
| 10.300 | 5.44391E+00 | 1.33718E-01 | -3.19838E-02 | 7.42511E+00 | -1.72196E-01 |
| 10.400 | 5.45712E+00 | 1.30558E-01 | -3.12329E-02 | 7.25483E+00 | -1.68386E-01 |
| 10.500 | 5.47002E+00 | 1.27471E-01 | -3.04995E-02 | 7.08831E+00 | -1.64655E-01 |
| 10.600 | 5.48261E+00 | 1.24457E-01 | -2.97832E-02 | 6.92549E+00 | -1.61000E-01 |
| 10.700 | 5.49491E+00 | 1.21514E-01 | -2.90836E-02 | 6.76629E+00 | -1.57420E-01 |
| 10.800 | 5.50692E+00 | 1.18640E-01 | -2.84002E-02 | 6.61062E+00 | -1.53915E-01 |
| 10.900 | 5.51864E+00 | 1.15833E-01 | -2.77327E-02 | 6.45843E+00 | -1.50482E-01 |
| 11.000 | 5.53009E+00 | 1.13093E-01 | -2.70808E-02 | 6.30964E+00 | -1.47121E-01 |
| 11.100 | 5.54126E+00 | 1.10417E-01 | -2.64440E-02 | 6.16417E+00 | -1.43830E-01 |
| 11.200 | 5.55217E+00 | 1.07803E-01 | -2.58222E-02 | 6.02195E+00 | -1.40609E-01 |
| 11.300 | 5.56283E+00 | 1.05252E-01 | -2.52148E-02 | 5.88293E+00 | -1.37455E-01 |
| 11.400 | 5.57323E+00 | 1.02760E-01 | -2.46215E-02 | 5.74702E+00 | -1.34367E-01 |
| 11.500 | 5.58338E+00 | 1.00327E-01 | -2.40422E-02 | 5.61417E+00 | -1.31345E-01 |
| 11.600 | 5.59329E+00 | 9.79512E-02 | -2.34763E-02 | 5.48431E+00 | -1.28387E-01 |
| 11.700 | 5.60297E+00 | 9.56313E-02 | -2.29237E-02 | 5.35737E+00 | -1.25493E-01 |
| 11.800 | 5.61242E+00 | 9.33660E-02 | -2.23839E-02 | 5.23330E+00 | -1.22666E-01 |
| 11.900 | 5.62165E+00 | 9.11541E-02 | -2.18568E-02 | 5.11204E+00 | -1.19887E-01 |
| 12.000 | 5.63065E+00 | 8.89942E-02 | -2.13420E-02 | 4.99351E+00 | -1.17174E-01 |
| 12.100 | 5.63945E+00 | 8.68835E-02 | -2.08393E-02 | 4.87767E+00 | -1.14520E-01 |
| 12.200 | 5.64803E+00 | 8.48260E-02 | -2.03483E-02 | 4.76445E+00 | -1.11923E-01 |
| 12.300 | 5.65641E+00 | 8.28152E-02 | -1.98688E-02 | 4.65380E+00 | -1.09381E-01 |
| 12.400 | 5.66460E+00 | 8.08519E-02 | -1.94005E-02 | 4.54567E+00 | -1.06895E-01 |
| 12.500 | 5.67259E+00 | 7.89348E-02 | -1.89431E-02 | 4.43999E+00 | -1.04463E-01 |
| 12.600 | 5.68039E+00 | 7.70629E-02 | -1.84965E-02 | 4.33672E+00 | -1.02084E-01 |
| 12.700 | 5.68800E+00 | 7.52351E-02 | -1.80604E-02 | 4.23581E+00 | -9.97572E-02 |
| 12.800 | 5.69543E+00 | 7.34505E-02 | -1.76345E-02 | 4.13719E+00 | -9.74809E-02 |

Pr = 0.0100 (Contd.)

| $\eta$ | F           | F'          | F''          | $\phi$       | $\phi'$      |
|--------|-------------|-------------|--------------|--------------|--------------|
| 12.900 | 5.70269E 00 | 7.17079E-02 | -1.72185E-02 | 4.04083E-01  | -9.52545E-02 |
| 13.000 | 5.70978E 00 | 7.00064E-02 | -1.68123E-02 | 3.94667E-01  | -9.30770E-02 |
| 13.100 | 5.71670E 00 | 6.83451E-02 | -1.64156E-02 | 3.85466E-01  | -9.09473E-02 |
| 13.200 | 5.72345E 00 | 6.67230E-02 | -1.60283E-02 | 3.76476E-01  | -8.86646E-02 |
| 13.300 | 5.73004E 00 | 6.51392E-02 | -1.56500E-02 | 3.67692E-01  | -8.68279E-02 |
| 13.400 | 5.73648E 00 | 6.35927E-02 | -1.52806E-02 | 3.59109E-01  | -8.48362E-02 |
| 13.500 | 5.74276E 00 | 6.20828E-02 | -1.49199E-02 | 3.50723E-01  | -8.28886E-02 |
| 13.600 | 5.74889E 00 | 6.06084E-02 | -1.45676E-02 | 3.42530E-01  | -8.09843E-02 |
| 13.700 | 5.75488E 00 | 5.91690E-02 | -1.42236E-02 | 3.34525E-01  | -7.91223E-02 |
| 13.800 | 5.76073E 00 | 5.77635E-02 | -1.38877E-02 | 3.26704E-01  | -7.73017E-02 |
| 13.900 | 5.76644E 00 | 5.63912E-02 | -1.35597E-02 | 3.19063E-01  | -7.55521E-02 |
| 14.000 | 5.77201E 00 | 5.50513E-02 | -1.32394E-02 | 3.11598E-01  | -7.37815E-02 |
| 14.100 | 5.77745E 00 | 5.37430E-02 | -1.29266E-02 | 3.04305E-01  | -7.20802E-02 |
| 14.200 | 5.78276E 00 | 5.24657E-02 | -1.26212E-02 | 2.97181E-01  | -7.04170E-02 |
| 14.300 | 5.78794E 00 | 5.12186E-02 | -1.23229E-02 | 2.90221E-01  | -6.87911E-02 |
| 14.400 | 5.79300E 00 | 5.00009E-02 | -1.20317E-02 | 2.83421E-01  | -6.72017E-02 |
| 14.500 | 5.79794E 00 | 4.88120E-02 | -1.17473E-02 | 2.76779E-01  | -6.56481E-02 |
| 14.600 | 5.80277E 00 | 4.76512E-02 | -1.14696E-02 | 2.70291E-01  | -6.41295E-02 |
| 14.700 | 5.80748E 00 | 4.65178E-02 | -1.11985E-02 | 2.63952E-01  | -6.26451E-02 |
| 14.800 | 5.81207E 00 | 4.54113E-02 | -1.09337E-02 | 2.57760E-01  | -6.11942E-02 |
| 14.900 | 5.81656E 00 | 4.43309E-02 | -1.06752E-02 | 2.51712E-01  | -5.97761E-02 |
| 15.000 | 5.82094E 00 | 4.32760E-02 | -1.04227E-02 | 2.45804E-01  | -5.83901E-02 |
| 15.100 | 5.82521E 00 | 4.22461E-02 | -1.01762E-02 | 2.40033E-01  | -5.70355E-02 |
| 15.200 | 5.82993E 00 | 4.12406E-02 | -9.93552E-03 | 2.34396E-01  | -5.57117E-02 |
| 15.300 | 5.83346E 00 | 4.02588E-02 | -9.70049E-03 | 2.28890E-01  | -5.44179E-02 |
| 15.400 | 5.83744E 00 | 3.93003E-02 | -9.47100E-03 | 2.23511E-01  | -5.31535E-02 |
| 15.500 | 5.84132E 00 | 3.83645E-02 | -9.24691E-03 | 2.18258E-01  | -5.19178E-02 |
| 15.600 | 5.84511E 00 | 3.74508E-02 | -9.02811E-03 | 2.13127E-01  | -5.07104E-02 |
| 15.700 | 5.84881E 00 | 3.65587E-02 | -8.81446E-03 | 2.08115E-01  | -4.95304E-02 |
| 15.800 | 5.85243E 00 | 3.56877E-02 | -8.60585E-03 | 2.03220E-01  | -4.83774E-02 |
| 15.900 | 5.85595E 00 | 3.48373E-02 | -8.40216E-03 | 1.98439E-01  | -4.72507E-02 |
| 16.000 | 5.85940E 00 | 3.40071E-02 | -8.20327E-03 | 1.93769E-01  | -4.61498E-02 |
| 16.100 | 5.86276E 00 | 3.31965E-02 | -8.00907E-03 | 1.89208E-01  | -4.50741E-02 |
| 16.200 | 5.86604E 00 | 3.24051E-02 | -7.81945E-03 | 1.84753E-01  | -4.40230E-02 |
| 16.300 | 5.86924E 00 | 3.16325E-02 | -7.63430E-03 | 1.80403E-01  | -4.29966E-02 |
| 16.400 | 5.87236E 00 | 3.08781E-02 | -7.45353E-03 | 1.76153E-01  | -4.19926E-02 |
| 16.500 | 5.87541E 00 | 3.01416E-02 | -7.27701E-03 | 1.72003E-01  | -4.10122E-02 |
| 16.600 | 5.87839E 00 | 2.94226E-02 | -7.10467E-03 | 1.67950E-01  | -4.00543E-02 |
| 16.700 | 5.88130E 00 | 2.87206E-02 | -6.93639E-03 | 1.63992E-01  | -3.91185E-02 |
| 16.800 | 5.88414E 00 | 2.80352E-02 | -6.77208E-03 | 1.60126E-01  | -3.82042E-02 |
| 16.900 | 5.88691E 00 | 2.73660E-02 | -6.61165E-03 | 1.56350E-01  | -3.73110E-02 |
| 17.000 | 5.88961E 00 | 2.67127E-02 | -6.45501E-03 | 1.526663E-01 | -3.64383E-02 |
| 17.100 | 5.89225E 00 | 2.60749E-02 | -6.30206E-03 | 1.49062E-01  | -3.55858E-02 |
| 17.200 | 5.89482E 00 | 2.54522E-02 | -6.15273E-03 | 1.45545E-01  | -3.47529E-02 |
| 17.300 | 5.89734E 00 | 2.48442E-02 | -6.00693E-03 | 1.42111E-01  | -3.39393E-02 |
| 17.400 | 5.89979E 00 | 2.42507E-02 | -5.84645E-03 | 1.38757E-01  | -3.31445E-02 |
| 17.500 | 5.90219E 00 | 2.36712E-02 | -5.72557E-03 | 1.35481E-01  | -3.23681E-02 |
| 17.600 | 5.90453E 00 | 2.31055E-02 | -5.58985E-03 | 1.32228E-01  | -3.16096E-02 |
| 17.700 | 5.90681E 00 | 2.25531E-02 | -5.45734E-03 | 1.29159E-01  | -3.08687E-02 |
| 17.800 | 5.90904E 00 | 2.20139E-02 | -5.32796E-03 | 1.26108E-01  | -3.01450E-02 |
| 17.900 | 5.91122E 00 | 2.14874E-02 | -5.20164E-03 | 1.23129E-01  | -2.94380E-02 |
| 18.000 | 5.91334E 00 | 2.09735E-02 | -5.07830E-03 | 1.20220E-01  | -2.87475E-02 |
| 18.100 | 5.91541E 00 | 2.04717E-02 | -4.95788E-03 | 1.17379E-01  | -2.80729E-02 |
| 18.200 | 5.91743E 00 | 1.99818E-02 | -4.84030E-03 | 1.14605E-01  | -2.74140E-02 |
| 18.300 | 5.91941E 00 | 1.95035E-02 | -4.72551E-03 | 1.11896E-01  | -2.67705E-02 |
| 18.400 | 5.92133E 00 | 1.90366E-02 | -4.61342E-03 | 1.09250E-01  | -2.61418E-02 |
| 18.500 | 5.92321E 00 | 1.85808E-02 | -4.50399E-03 | 1.06667E-01  | -2.55278E-02 |
| 18.600 | 5.92505E 00 | 1.81357E-02 | -4.39714E-03 | 1.04144E-01  | -2.49281E-02 |
| 18.700 | 5.92684E 00 | 1.77013E-02 | -4.29282E-03 | 1.01681E-01  | -2.43423E-02 |
| 18.800 | 5.92859E 00 | 1.72771E-02 | -4.19079E-03 | 9.92752E-02  | -2.37702E-02 |
| 18.900 | 5.93030E 00 | 1.68630E-02 | -4.09152E-03 | 9.69262E-02  | -2.32114E-02 |
| 19.000 | 5.93196E 00 | 1.64587E-02 | -3.99443E-03 | 9.46325E-02  | -2.26656E-02 |
| 19.100 | 5.93359E 00 | 1.60640E-02 | -3.89963E-03 | 9.23926E-02  | -2.21326E-02 |
| 19.200 | 5.93518E 00 | 1.56787E-02 | -3.80707E-03 | 9.02055E-02  | -2.16120E-02 |
| 19.300 | 5.93673E 00 | 1.53025E-02 | -3.71671E-03 | 8.80698E-02  | -2.11035E-02 |
| 19.400 | 5.93824E 00 | 1.49535E-02 | -3.62848E-03 | 8.59844E-02  | -2.06069E-02 |
| 19.500 | 5.93971E 00 | 1.45768E-02 | -3.54233E-03 | 8.39481E-02  | -2.01219E-02 |
| 19.600 | 5.94115E 00 | 1.42268E-02 | -3.45823E-03 | 8.19597E-02  | -1.96482E-02 |
| 19.700 | 5.94256E 00 | 1.38851E-02 | -3.37611E-03 | 8.00181E-02  | -1.91856E-02 |

**Pr = 0.0100 (Contd.)**

| $\eta$ | F           | F'          | F''          | $\phi$      | $\phi'$      |
|--------|-------------|-------------|--------------|-------------|--------------|
| 19.800 | 5.94393E 00 | 1.35515E-02 | -3.29594E-03 | 7.81222E-02 | -1.87338E-02 |
| 19.900 | 5.94527E 00 | 1.32258E-02 | -3.21767E-03 | 7.62710E-02 | -1.82926E-02 |
| 20.000 | 5.94658E 00 | 1.29079E-02 | -3.14124E-03 | 7.44633E-02 | -1.78617E-02 |
| 20.100 | 5.94785E 00 | 1.25975E-02 | -3.06663E-03 | 7.26983E-02 | -1.74409E-02 |
| 20.200 | 5.94910E 00 | 1.22945E-02 | -2.99378E-03 | 7.09758E-02 | -1.70299E-02 |
| 20.300 | 5.95031E 00 | 1.19987E-02 | -2.92266E-03 | 6.92920E-02 | -1.66285E-02 |
| 20.400 | 5.95150E 00 | 1.17099E-02 | -2.85321E-03 | 6.76488E-02 | -1.62366E-02 |
| 20.500 | 5.95265E 00 | 1.14280E-02 | -2.78542E-03 | 6.60444E-02 | -1.58538E-02 |
| 20.600 | 5.95378E 00 | 1.11528E-02 | -2.71922E-03 | 6.44777E-02 | -1.5480CE-02 |
| 20.700 | 5.95488E 00 | 1.08841E-02 | -2.65460E-03 | 6.29481E-02 | -1.51150E-02 |
| 20.800 | 5.95596E 00 | 1.06218E-02 | -2.59150E-03 | 6.14544E-02 | -1.47585E-02 |
| 20.900 | 5.95701E 00 | 1.03657E-02 | -2.52989E-03 | 5.99961E-02 | -1.44104E-02 |
| 21.000 | 5.95803E 00 | 1.01158E-02 | -2.46975E-03 | 5.85721E-02 | -1.40705E-02 |
| 21.100 | 5.95903E 00 | 9.87174E-03 | -2.41103E-03 | 5.71817E-02 | -1.37385E-02 |
| 21.200 | 5.96001E 00 | 9.63351E-03 | -2.35370E-03 | 5.58241E-02 | -1.34143E-02 |
| 21.300 | 5.96096E 00 | 9.40095E-03 | -2.29772E-03 | 5.44986E-02 | -1.30978E-02 |
| 21.400 | 5.96189E 00 | 9.17393E-03 | -2.24307E-03 | 5.32043E-02 | -1.27887E-02 |
| 21.500 | 5.96279E 00 | 8.95230E-03 | -2.18972E-03 | 5.19406E-02 | -1.24868E-02 |
| 21.600 | 5.96368E 00 | 8.73594E-03 | -2.13763E-03 | 5.07067E-02 | -1.21920E-02 |
| 21.700 | 5.96454E 00 | 8.52473E-03 | -2.08677E-03 | 4.95020E-02 | -1.19042E-02 |
| 21.800 | 5.96538E 00 | 8.31855E-03 | -2.03712E-03 | 4.83257E-02 | -1.16231E-02 |
| 21.900 | 5.96620E 00 | 8.11727E-03 | -1.98864E-03 | 4.71771E-02 | -1.13486E-02 |
| 22.000 | 5.96701E 00 | 7.92078E-03 | -1.94131E-03 | 4.60557E-02 | -1.10806E-02 |
| 22.100 | 5.96779E 00 | 7.72897E-03 | -1.89510E-03 | 4.49608E-02 | -1.08189E-02 |
| 22.200 | 5.96855E 00 | 7.54172E-03 | -1.84998E-03 | 4.38917E-02 | -1.05634E-02 |
| 22.300 | 5.96930E 00 | 7.35894E-03 | -1.80594E-03 | 4.28479E-02 | -1.03138E-02 |
| 22.400 | 5.97002E 00 | 7.18050E-03 | -1.76293E-03 | 4.18288E-02 | -1.00702E-02 |
| 22.500 | 5.97073E 00 | 7.00632E-03 | -1.72095E-03 | 4.08337E-02 | -9.83222E-03 |
| 22.600 | 5.97143E 00 | 6.83628E-03 | -1.67996E-03 | 3.98621E-02 | -9.59989E-03 |
| 22.700 | 5.97210E 00 | 6.67029E-03 | -1.63994E-03 | 3.89135E-02 | -9.37303E-03 |
| 22.800 | 5.97276E 00 | 6.50826E-03 | -1.60087E-03 | 3.79874E-02 | -9.15152E-03 |
| 22.900 | 5.97340E 00 | 6.35009E-03 | -1.56272E-03 | 3.70831E-02 | -8.93522E-03 |
| 23.000 | 5.97403E 00 | 6.19569E-03 | -1.52548E-03 | 3.62001E-02 | -8.72402E-03 |
| 23.100 | 5.97464E 00 | 6.04496E-03 | -1.48912E-03 | 3.53381E-02 | -8.51779E-03 |
| 23.200 | 5.97524E 00 | 5.89783E-03 | -1.45362E-03 | 3.44964E-02 | -8.31642E-03 |
| 23.300 | 5.97582E 00 | 5.75421E-03 | -1.41896E-03 | 3.36747E-02 | -8.11980E-03 |
| 23.400 | 5.97639E 00 | 5.61402E-03 | -1.38512E-03 | 3.28723E-02 | -7.92782E-03 |
| 23.500 | 5.97694E 00 | 5.47716E-03 | -1.35209E-03 | 3.20889E-02 | -7.74036E-03 |
| 23.600 | 5.97749E 00 | 5.34357E-03 | -1.31983E-03 | 3.13241E-02 | -7.55732E-03 |
| 23.700 | 5.97801E 00 | 5.21317E-03 | -1.28834E-03 | 3.05773E-02 | -7.37860E-03 |
| 23.800 | 5.97853E 00 | 5.08588E-03 | -1.25760E-03 | 2.98482E-02 | -7.20410E-03 |
| 23.900 | 5.97903E 00 | 4.96163E-03 | -1.22759E-03 | 2.91364E-02 | -7.03371E-03 |
| 24.000 | 5.97952E 00 | 4.84034E-03 | -1.19828E-03 | 2.84414E-02 | -6.86734E-03 |
| 24.100 | 5.98000E 00 | 4.72195E-03 | -1.16967E-03 | 2.77628E-02 | -6.70489E-03 |
| 24.200 | 5.98047E 00 | 4.60638E-03 | -1.14174E-03 | 2.71003E-02 | -6.54628E-03 |
| 24.300 | 5.98092E 00 | 4.49358E-03 | -1.11447E-03 | 2.64534E-02 | -6.39142E-03 |
| 24.400 | 5.98136E 00 | 4.38347E-03 | -1.08785E-03 | 2.58218E-02 | -6.24020E-03 |
| 24.500 | 5.98180E 00 | 4.27599E-03 | -1.06186E-03 | 2.52052E-02 | -6.09256E-03 |
| 24.600 | 5.98222E 00 | 4.17107E-03 | -1.03648E-03 | 2.46032E-02 | -5.94844E-03 |
| 24.700 | 5.98263E 00 | 4.06867E-03 | -1.01170E-03 | 2.40154E-02 | -5.80765E-03 |
| 24.800 | 5.98303E 00 | 3.96871E-03 | -9.87514E-04 | 2.34416E-02 | -5.67022E-03 |
| 24.900 | 5.98343E 00 | 3.87115E-03 | -9.63899E-04 | 2.28813E-02 | -5.53604E-03 |
| 25.000 | 5.98381E 00 | 3.77592E-03 | -9.40843E-04 | 2.23343E-02 | -5.40520E-03 |
| 25.100 | 5.98418E 00 | 3.68296E-03 | -9.18334E-04 | 2.18002E-02 | -5.27711E-03 |
| 25.200 | 5.98454E 00 | 3.59223E-03 | -8.96358E-04 | 2.12787E-02 | -5.15221E-03 |
| 25.300 | 5.98490E 00 | 3.50367E-03 | -8.74902E-04 | 2.07696E-02 | -5.03266E-03 |
| 25.400 | 5.98524E 00 | 3.41723E-03 | -8.53955E-04 | 2.02726E-02 | -4.91119E-03 |
| 25.500 | 5.98558E 00 | 3.33286E-03 | -8.33505E-04 | 1.97873E-02 | -4.79494E-03 |
| 25.600 | 5.98591E 00 | 3.25052E-03 | -8.13539E-04 | 1.93135E-02 | -4.68144E-03 |
| 25.700 | 5.98623E 00 | 3.17014E-03 | -7.94046E-04 | 1.88509E-02 | -4.57061E-03 |
| 25.800 | 5.98655E 00 | 3.09169E-03 | -7.75016E-04 | 1.83993E-02 | -4.46241E-03 |
| 25.900 | 5.98685E 00 | 3.01512E-03 | -7.56436E-04 | 1.79584E-02 | -4.35676E-03 |
| 26.000 | 5.98715E 00 | 2.94039E-03 | -7.38296E-04 | 1.75279E-02 | -4.25362E-03 |
| 26.100 | 5.98744E 00 | 2.86745E-03 | -7.20587E-04 | 1.71076E-02 | -4.15290E-03 |
| 26.200 | 5.98772E 00 | 2.79626E-03 | -7.03297E-04 | 1.66972E-02 | -4.05458E-03 |
| 26.300 | 5.98800E 00 | 2.72678E-03 | -6.86417E-04 | 1.62966E-02 | -3.95857E-03 |
| 26.400 | 5.98827E 00 | 2.65896E-03 | -6.69937E-04 | 1.59054E-02 | -3.86484E-03 |
| 26.500 | 5.98853E 00 | 2.59278E-03 | -6.53847E-04 | 1.55235E-02 | -3.77332E-03 |
| 26.600 | 5.98879E 00 | 2.52818E-03 | -6.38139E-04 | 1.51507E-02 | -3.68397E-03 |

Pr = 0.0100 (Contd.)

| $\eta$ | F           | F'          | F''          | $\phi$      | $\phi'$      |
|--------|-------------|-------------|--------------|-------------|--------------|
| 26.700 | 5.98904E+00 | 2.46514E-03 | -6.22803E-04 | 1.47867E-02 | -3.59673E-03 |
| 26.800 | 5.98928E+00 | 2.40361E-03 | -6.07831E-04 | 1.44313E-02 | -3.51155E-03 |
| 26.900 | 5.98952E+00 | 2.34356E-03 | -5.93213E-04 | 1.40843E-02 | -3.42839E-03 |
| 27.000 | 5.98975E+00 | 2.28495E-03 | -5.78942E-04 | 1.37455E-02 | -3.34719E-03 |
| 27.100 | 5.98997E+00 | 2.22776E-03 | -5.65009E-04 | 1.34148E-02 | -3.26792E-03 |
| 27.200 | 5.99019E+00 | 2.17194E-03 | -5.51407E-04 | 1.30919E-02 | -3.19052E-03 |
| 27.300 | 5.99041E+00 | 2.11747E-03 | -5.38126E-04 | 1.27766E-02 | -3.11495E-03 |
| 27.400 | 5.99062E+00 | 2.06430E-03 | -5.25161E-04 | 1.24688E-02 | -3.04117E-03 |
| 27.500 | 5.99082E+00 | 2.01242E-03 | -5.12503E-04 | 1.21683E-02 | -2.96914E-03 |
| 27.600 | 5.99102E+00 | 1.96179E-03 | -5.00145E-04 | 1.18749E-02 | -2.89881E-03 |
| 27.700 | 5.99121E+00 | 1.91238E-03 | -4.88079E-04 | 1.15885E-02 | -2.83015L-03 |
| 27.800 | 5.99140E+00 | 1.86417E-03 | -4.76300E-04 | 1.13089E-02 | -2.76310E-03 |
| 27.900 | 5.99159E+00 | 1.81712E-03 | -4.64800E-04 | 1.10358E-02 | -2.69765E-03 |
| 28.000 | 5.99177E+00 | 1.77120E-03 | -4.53573E-04 | 1.07693E-02 | -2.63357E-03 |
| 28.100 | 5.99194E+00 | 1.72639E-03 | -4.42611E-04 | 1.05090E-02 | -2.57135E-03 |
| 28.200 | 5.99211E+00 | 1.68267E-03 | -4.31910E-04 | 1.02550E-02 | -2.51044E-03 |
| 28.300 | 5.99228E+00 | 1.64000E-03 | -4.21462E-04 | 1.00069E-02 | -2.45097E-03 |
| 28.400 | 5.99244E+00 | 1.59837E-03 | -4.11262E-04 | 9.76472E-03 | -2.39290E-03 |
| 28.500 | 5.99260E+00 | 1.55774E-03 | -4.01303E-04 | 9.52827E-03 | -2.33621E-03 |
| 28.600 | 5.99275E+00 | 1.51810E-03 | -3.91581E-04 | 9.29743E-03 | -2.28086E-03 |
| 28.700 | 5.99290E+00 | 1.47942E-03 | -3.82089E-04 | 9.07206E-03 | -2.22628E-03 |
| 28.800 | 5.99305E+00 | 1.44167E-03 | -3.72822E-04 | 8.85202E-03 | -2.17406E-03 |
| 28.900 | 5.99319E+00 | 1.40485E-03 | -3.63774E-04 | 8.63720E-03 | -2.12255E-03 |
| 29.000 | 5.99333E+00 | 1.36891E-03 | -3.54942E-04 | 8.42747E-03 | -2.07226E-03 |
| 29.100 | 5.99346E+00 | 1.33385E-03 | -3.46318E-04 | 8.22271E-03 | -2.02316E-03 |
| 29.200 | 5.99359E+00 | 1.29964E-03 | -3.37898E-04 | 8.02280E-03 | -1.97522E-03 |
| 29.300 | 5.99372E+00 | 1.26626E-03 | -3.29680E-04 | 7.82763E-03 | -1.92842E-03 |
| 29.400 | 5.99385E+00 | 1.23370E-03 | -3.21655E-04 | 7.63708E-03 | -1.88273E-03 |
| 29.500 | 5.99397E+00 | 1.20193E-03 | -3.13821E-04 | 7.45105E-03 | -1.83812E-03 |
| 29.600 | 5.99409E+00 | 1.17093E-03 | -3.06172E-04 | 7.26942E-03 | -1.79456E-03 |
| 29.700 | 5.99420E+00 | 1.14069E-03 | -2.98704E-04 | 7.09210E-03 | -1.75204E-03 |
| 29.800 | 5.99432E+00 | 1.11118E-03 | -2.91414E-04 | 6.91898E-03 | -1.71052E-03 |
| 29.900 | 5.99443E+00 | 1.08240E-03 | -2.84296E-04 | 6.74996E-03 | -1.66999E-03 |
| 30.000 | 5.99453E+00 | 1.05432E-03 | -2.77348E-04 | 6.58495E-03 | -1.63041E-03 |
| 30.100 | 5.99464E+00 | 1.02692E-03 | -2.70563E-04 | 6.42385E-03 | -1.59178E-03 |
| 30.200 | 5.99474E+00 | 1.00020E-03 | -2.63940E-04 | 6.26657E-03 | -1.55460E-03 |
| 30.300 | 5.99484E+00 | 9.74129E-04 | -2.57474E-04 | 6.11301E-03 | -1.51723E-03 |
| 30.400 | 5.99493E+00 | 9.48699E-04 | -2.51161E-04 | 5.96309E-03 | -1.48127E-03 |
| 30.500 | 5.99503E+00 | 9.23892E-04 | -2.44997E-04 | 5.81673E-03 | -1.44617E-03 |
| 30.600 | 5.99512E+00 | 8.99695E-04 | -2.38980E-04 | 5.67383E-03 | -1.41190E-03 |
| 30.700 | 5.99521E+00 | 8.76092E-04 | -2.33105E-04 | 5.53432E-03 | -1.37844E-03 |
| 30.800 | 5.99529E+00 | 8.53069E-04 | -2.27370E-04 | 5.39812E-03 | -1.34577E-03 |
| 30.900 | 5.99538E+00 | 8.30613E-04 | -2.21771E-04 | 5.26514E-03 | -1.31387E-03 |
| 31.000 | 5.99546E+00 | 8.08710E-04 | -2.16304E-04 | 5.13532E-03 | -1.28274E-03 |
| 31.100 | 5.99554E+00 | 7.87348E-04 | -2.10967E-04 | 5.00857E-03 | -1.25234E-03 |
| 31.200 | 5.99562E+00 | 7.66513E-04 | -2.05756E-04 | 4.88483E-03 | -1.22266E-03 |
| 31.300 | 5.99569E+00 | 7.46192E-04 | -2.00669E-04 | 4.76401E-03 | -1.19368E-03 |
| 31.400 | 5.99577E+00 | 7.26375E-04 | -1.95703E-04 | 4.64607E-03 | -1.16539E-03 |
| 31.500 | 5.99584E+00 | 7.07048E-04 | -1.90854E-04 | 4.53091E-03 | -1.13777E-03 |
| 31.600 | 5.99591E+00 | 6.88200E-04 | -1.86120E-04 | 4.41849E-03 | -1.11080E-03 |
| 31.700 | 5.99598E+00 | 6.69820E-04 | -1.81499E-04 | 4.30873E-03 | -1.08447E-03 |
| 31.800 | 5.99604E+00 | 6.51897E-04 | -1.76987E-04 | 4.20158E-03 | -1.05877E-03 |
| 31.900 | 5.99611E+00 | 6.34419E-04 | -1.72582E-04 | 4.09696E-03 | -1.03368E-03 |
| 32.000 | 5.99617E+00 | 6.17377E-04 | -1.68281E-04 | 3.99482E-03 | -1.00918E-03 |
| 32.100 | 5.99623E+00 | 6.00760E-04 | -1.64083E-04 | 3.89510E-03 | -9.85257E-04 |
| 32.200 | 5.99629E+00 | 5.84557E-04 | -1.59983E-04 | 3.79775E-03 | -9.61904E-04 |
| 32.300 | 5.99635E+00 | 5.68760E-04 | -1.55981E-04 | 3.70271E-03 | -9.39105E-04 |
| 32.400 | 5.99640E+00 | 5.53358E-04 | -1.52074E-04 | 3.60991E-03 | -9.16846E-04 |
| 32.500 | 5.99646E+00 | 5.38342E-04 | -1.48260E-04 | 3.51932E-03 | -8.95115E-04 |
| 32.600 | 5.99651E+00 | 5.23703E-04 | -1.44536E-04 | 3.43087E-03 | -8.73898E-04 |
| 32.700 | 5.99656E+00 | 5.09431E-04 | -1.40409E-04 | 3.34452E-03 | -8.53184E-04 |
| 32.800 | 5.99661E+00 | 4.95520E-04 | -1.37351E-04 | 3.26022E-03 | -8.32961E-04 |
| 32.900 | 5.99666E+00 | 4.81959E-04 | -1.33885E-04 | 3.17791E-03 | -8.13218E-04 |
| 33.000 | 5.99671E+00 | 4.68740E-04 | -1.30502E-04 | 3.09756E-03 | -7.93942E-04 |
| 33.100 | 5.99675E+00 | 4.55856E-04 | -1.27199E-04 | 3.01911E-03 | -7.75123E-04 |
| 33.200 | 5.99680E+00 | 4.43297E-04 | -1.23974E-04 | 2.94252E-03 | -7.56750E-04 |
| 33.300 | 5.99684E+00 | 4.31058E-04 | -1.20826E-04 | 2.86775E-03 | -7.38812E-04 |
| 33.400 | 5.99689E+00 | 4.19130E-04 | -1.17752E-04 | 2.79474E-03 | -7.21300E-04 |
| 33.500 | 5.99693E+00 | 4.07505E-04 | -1.14751E-04 | 2.72347E-03 | -7.04202E-04 |

**Pr = 0.0100 (Contd.)**

| $\eta$ | F           | F'          | F''          | $\phi$      | $\psi$       |
|--------|-------------|-------------|--------------|-------------|--------------|
| 33.600 | 5.99697E 00 | 3.96177E-04 | -1.11822E-04 | 2.65389E-03 | -6.87510E-04 |
| 33.700 | 5.99701E 00 | 3.85139E-04 | -1.08962E-04 | 2.58596E-03 | -6.71213E-04 |
| 33.800 | 5.99704E 00 | 3.74383E-04 | -1.06169E-04 | 2.51963E-03 | -6.55303E-04 |
| 33.900 | 5.99708E 00 | 3.63903E-04 | -1.03443E-04 | 2.45488E-03 | -6.39769E-04 |
| 34.000 | 5.99712E 00 | 3.53692E-04 | -1.00781E-04 | 2.39167E-03 | -6.24604E-04 |
| 34.100 | 5.99715E 00 | 3.43744E-04 | -9.81829E-05 | 2.32995E-03 | -6.09798E-04 |
| 34.200 | 5.99719E 00 | 3.34053E-04 | -9.56460E-05 | 2.26970E-03 | -5.95343E-04 |
| 34.300 | 5.99722E 00 | 3.24613E-04 | -9.31693E-05 | 2.21087E-03 | -5.81231E-04 |
| 34.400 | 5.99725E 00 | 3.15417E-04 | -9.07513E-05 | 2.15344E-03 | -5.67453E-04 |
| 34.500 | 5.99728E 00 | 3.06461E-04 | -8.83906E-05 | 2.09737E-03 | -5.54001E-04 |
| 34.600 | 5.99731E 00 | 2.97737E-04 | -8.60858E-05 | 2.04263E-03 | -5.40869E-04 |
| 34.700 | 5.99734E 00 | 2.89242E-04 | -8.38358E-05 | 1.98919E-03 | -5.28048E-04 |
| 34.800 | 5.99737E 00 | 2.80969E-04 | -8.16390E-05 | 1.93701E-03 | -5.1553CE-04 |
| 34.900 | 5.99740E 00 | 2.72912E-04 | -7.94943E-05 | 1.88607E-03 | -5.03131E-04 |
| 35.000 | 5.99742E 00 | 2.65068E-04 | -7.74005E-05 | 1.83634E-03 | -4.91379E-04 |
| 35.100 | 5.99745E 00 | 2.57431E-04 | -7.53563E-05 | 1.78778E-03 | -4.79730E-04 |
| 35.200 | 5.99748E 00 | 2.49995E-04 | -7.33606E-05 | 1.74038E-03 | -4.68358E-04 |
| 35.300 | 5.99750E 00 | 2.42757E-04 | -7.14222E-05 | 1.69410L-03 | -4.57256E-04 |
| 35.400 | 5.99752E 00 | 2.35711E-04 | -6.95100E-05 | 1.64892E-03 | -4.46416E-04 |
| 35.500 | 5.99755E 00 | 2.28853E-04 | -6.76528E-05 | 1.60481E-03 | -4.35834E-04 |
| 35.600 | 5.99757E 00 | 2.22179E-04 | -6.58397E-05 | 1.56175E-03 | -4.25502E-04 |
| 35.700 | 5.99759E 00 | 2.15684E-04 | -6.40696E-05 | 1.51970E-03 | -4.15415E-04 |
| 35.800 | 5.99761E 00 | 2.09364E-04 | -6.23415E-05 | 1.47866E-03 | -4.05567E-04 |
| 35.900 | 5.99763E 00 | 2.03219E-04 | -6.06543E-05 | 1.43858E-03 | -3.95953E-04 |
| 36.000 | 5.99765E 00 | 1.97232E-04 | -5.90071E-05 | 1.39946E-03 | -3.86567E-04 |
| 36.100 | 5.99767E 00 | 1.91412E-04 | -5.73990E-05 | 1.36126E-03 | -3.77403E-04 |
| 36.200 | 5.99769E 00 | 1.85750E-04 | -5.58289E-05 | 1.32397E-03 | -3.68456E-04 |
| 36.300 | 5.99771E 00 | 1.80245E-04 | -5.42962E-05 | 1.28753E-03 | -3.59722E-04 |
| 36.400 | 5.99773E 00 | 1.74890E-04 | -5.27997E-05 | 1.25202E-03 | -3.51194E-04 |
| 36.500 | 5.99775E 00 | 1.69683E-04 | -5.13387E-05 | 1.21732E-03 | -3.42869E-04 |
| 36.600 | 5.99776E 00 | 1.64621E-04 | -4.99124E-05 | 1.18344E-03 | -3.34741E-04 |
| 36.700 | 5.99778E 00 | 1.59700E-04 | -4.85199E-05 | 1.15036E-03 | -3.26050E-04 |
| 36.800 | 5.99779E 00 | 1.54916E-04 | -4.71603E-05 | 1.11807E-03 | -3.19058E-04 |
| 36.900 | 5.99781E 00 | 1.50267E-04 | -4.58331E-05 | 1.08655E-03 | -3.11494E-04 |
| 37.000 | 5.99782E 00 | 1.45748E-04 | -4.45372E-05 | 1.05577E-03 | -3.04101E-04 |
| 37.100 | 5.99784E 00 | 1.41358E-04 | -4.32721E-05 | 1.02572E-03 | -2.96960E-04 |
| 37.200 | 5.99785E 00 | 1.37093E-04 | -4.20370E-05 | 9.96381E-04 | -2.89862E-04 |
| 37.300 | 5.99787E 00 | 1.32950E-04 | -4.08312E-05 | 9.67790E-04 | -2.82990E-04 |
| 37.400 | 5.99788E 00 | 1.28926E-04 | -3.96540E-05 | 9.39777E-04 | -2.76282E-04 |
| 37.500 | 5.99789E 00 | 1.25018E-04 | -3.85047E-05 | 9.12478E-04 | -2.69732E-04 |
| 37.600 | 5.99790E 00 | 1.21224E-04 | -3.73826E-05 | 8.85826E-04 | -2.63338E-04 |
| 37.700 | 5.99792E 00 | 1.17541E-04 | -3.62871E-05 | 8.59805E-04 | -2.57095E-04 |
| 37.800 | 5.99793E 00 | 1.13966E-04 | -3.52176E-05 | 8.34402E-04 | -2.51000E-04 |
| 37.900 | 5.99794E 00 | 1.10496E-04 | -3.41734E-05 | 8.09600E-04 | -2.45050E-04 |
| 38.000 | 5.99795E 00 | 1.07130E-04 | -3.31540E-05 | 7.85387E-04 | -2.39242E-04 |
| 38.100 | 5.99796E 00 | 1.03865E-04 | -3.21588E-05 | 7.61748E-04 | -2.33569E-04 |
| 38.200 | 5.99797E 00 | 1.00698E-04 | -3.11872E-05 | 7.38669E-04 | -2.28032E-04 |
| 38.300 | 5.99798E 00 | 9.76265E-05 | -3.02386E-05 | 7.16137E-04 | -2.22626E-04 |
| 38.400 | 5.99799E 00 | 9.46491E-05 | -2.93124E-05 | 6.94139E-04 | -2.17348E-04 |
| 38.500 | 5.99800E 00 | 9.17633E-05 | -2.84083E-05 | 6.72663E-04 | -2.12195E-04 |
| 38.600 | 5.99801E 00 | 8.89668E-05 | -2.75256E-05 | 6.51696E-04 | -2.07165E-04 |
| 38.700 | 5.99802E 00 | 8.62575E-05 | -2.66638E-05 | 6.31226E-04 | -2.02254E-04 |
| 38.800 | 5.99803E 00 | 8.36333E-05 | -2.58224E-05 | 6.11242E-04 | -1.97459E-04 |
| 38.900 | 5.99803E 00 | 8.10923E-05 | -2.50010E-05 | 5.91731E-04 | -1.92778E-04 |
| 39.000 | 5.99804E 00 | 7.86325E-05 | -2.41990E-05 | 5.72682E-04 | -1.88028E-04 |
| 39.100 | 5.99805E 00 | 7.62519E-05 | -2.34161E-05 | 5.54086E-04 | -1.83746E-04 |
| 39.200 | 5.99806E 00 | 7.39487E-05 | -2.26517E-05 | 5.35930E-04 | -1.79390E-04 |
| 39.300 | 5.99806E 00 | 7.17209E-05 | -2.19055E-05 | 5.18204E-04 | -1.75137E-04 |
| 39.400 | 5.99807E 00 | 6.95670E-05 | -2.11769E-05 | 5.00899E-04 | -1.70985E-04 |
| 39.500 | 5.99808E 00 | 6.74850E-05 | -2.04656E-05 | 4.84004E-04 | -1.66932L-04 |
| 39.600 | 5.99808E 00 | 6.54733E-05 | -1.97712E-05 | 4.67509E-04 | -1.62974E-04 |
| 39.700 | 5.99809E 00 | 6.35302E-05 | -1.90933E-05 | 4.51406E-04 | -1.59110E-04 |
| 39.800 | 5.99810E 00 | 6.16541E-05 | -1.84314E-05 | 4.35684E-04 | -1.55338E-04 |
| 39.900 | 5.99810E 00 | 5.98434E-05 | -1.77852E-05 | 4.20335E-04 | -1.51656E-04 |
| 40.000 | 5.99811E 00 | 5.80965E-05 | -1.71543E-05 | 4.05350E-04 | -1.48061E-04 |
| 40.100 | 5.99812E 00 | 5.64120E-05 | -1.65338E-05 | 3.90720E-04 | -1.44550E-04 |
| 40.200 | 5.99812E 00 | 5.47884E-05 | -1.59371E-05 | 3.76437E-04 | -1.41124E-04 |
| 40.300 | 5.99813E 00 | 5.32241E-05 | -1.53500E-05 | 3.62493E-04 | -1.37778E-04 |
| 40.400 | 5.99813E 00 | 5.17179E-05 | -1.47769E-05 | 3.48879E-04 | -1.34512E-04 |

$Pr = 0.0100$  (Contd.)

| $\eta$ | $F$         | $F'$        | $F''$        | $\phi$      | $\phi'$      |
|--------|-------------|-------------|--------------|-------------|--------------|
| 40.500 | 5.99814E 00 | 5.02683E-05 | -1.42173E-05 | 3.35588E-04 | -1.31323E-04 |
| 40.600 | 5.99814E 00 | 4.88740E-05 | -1.36710E-05 | 3.22612E-04 | -1.28209E-04 |
| 40.700 | 5.99815E 00 | 4.75337E-05 | -1.31377E-05 | 3.09944E-04 | -1.25170E-04 |
| 40.800 | 5.99815E 00 | 4.62461E-05 | -1.26170E-05 | 2.97576E-04 | -1.22203E-04 |
| 40.900 | 5.99816E 00 | 4.50099E-05 | -1.21086E-05 | 2.85501E-04 | -1.19305E-04 |
| 41.000 | 5.99816E 00 | 4.38239E-05 | -1.16123E-05 | 2.73712E-04 | -1.16477E-04 |
| 41.100 | 5.99816E 00 | 4.26870E-05 | -1.11278E-05 | 2.62203E-04 | -1.13716E-04 |
| 41.200 | 5.99817E 00 | 4.15980E-05 | -1.06547E-05 | 2.50967E-04 | -1.11020E-04 |
| 41.300 | 5.99817E 00 | 4.05557E-05 | -1.01929E-05 | 2.39997E-04 | -1.08388E-04 |
| 41.400 | 5.99818E 00 | 3.95590E-05 | -9.74203E-06 | 2.29287E-04 | -1.05818E-04 |
| 41.500 | 5.99818E 00 | 3.86069E-05 | -9.30183E-06 | 2.18831E-04 | -1.03310E-04 |
| 41.600 | 5.99818E 00 | 3.76983E-05 | -8.87208E-06 | 2.08623E-04 | -1.00860E-04 |
| 41.700 | 5.99819E 00 | 3.68322E-05 | -8.45251E-06 | 1.98657E-04 | -9.84693E-05 |
| 41.800 | 5.99819E 00 | 3.60075E-05 | -8.04288E-06 | 1.88928E-04 | -9.61349E-05 |
| 41.900 | 5.99820E 00 | 3.52233E-05 | -7.64297E-06 | 1.79429E-04 | -9.38558E-05 |
| 42.000 | 5.99820E 00 | 3.44786E-05 | -7.25254E-06 | 1.70155E-04 | -9.16307E-05 |
| 42.100 | 5.99820E 00 | 3.37725E-05 | -6.87137E-06 | 1.61101E-04 | -8.94584E-05 |
| 42.200 | 5.99821E 00 | 3.31040E-05 | -6.49923E-06 | 1.52261E-04 | -8.73376E-05 |
| 42.300 | 5.99821E 00 | 3.24723E-05 | -6.13592E-06 | 1.43632E-04 | -8.52670E-05 |
| 42.400 | 5.99821E 00 | 3.18765E-05 | -5.78122E-06 | 1.35206E-04 | -8.32456E-05 |
| 42.500 | 5.99822E 00 | 3.13158E-05 | -5.43493E-06 | 1.26981E-04 | -8.12720E-05 |
| 42.600 | 5.99822E 00 | 3.07893E-05 | -5.09684E-06 | 1.18950E-04 | -7.93453E-05 |
| 42.700 | 5.99822E 00 | 3.02962E-05 | -4.76678E-06 | 1.11110E-04 | -7.74642E-05 |
| 42.800 | 5.99822E 00 | 2.98357E-05 | -4.44453E-06 | 1.03456E-04 | -7.56277E-05 |
| 42.900 | 5.99823E 00 | 2.94070E-05 | -4.12993E-06 | 9.59832E-05 | -7.38348E-05 |
| 43.000 | 5.99823E 00 | 2.90094E-05 | -3.82279E-06 | 8.86876E-05 | -7.20844E-05 |
| 43.100 | 5.99823E 00 | 2.86422E-05 | -3.52293E-06 | 8.15650E-05 | -7.03754E-05 |
| 43.200 | 5.99824E 00 | 2.83046E-05 | -3.23017E-06 | 7.46112E-05 | -6.87070E-05 |
| 43.300 | 5.99824E 00 | 2.79959E-05 | -2.94436E-06 | 6.78223E-05 | -6.70781E-05 |
| 43.400 | 5.99824E 00 | 2.77115E-05 | -2.66532E-06 | 6.11943E-05 | -6.54879E-05 |
| 43.500 | 5.99824E 00 | 2.74627E-05 | -2.39290E-06 | 5.47234E-05 | -6.39353E-05 |
| 43.600 | 5.99825E 00 | 2.72367E-05 | -2.12694E-06 | 4.84060E-05 | -6.24196E-05 |
| 43.700 | 5.99825E 00 | 2.70371E-05 | -1.86728E-06 | 4.22383E-05 | -6.09398E-05 |
| 43.800 | 5.99825E 00 | 2.68631E-05 | -1.61378E-06 | 3.62168E-05 | -5.94951E-05 |
| 43.900 | 5.99826E 00 | 2.67141E-05 | -1.36629E-06 | 3.03381E-05 | -5.80846E-05 |
| 44.000 | 5.99826E 00 | 2.65896E-05 | -1.12466E-06 | 2.45988E-05 | -5.67075E-05 |
| 44.100 | 5.99826E 00 | 2.64890E-05 | -8.88767E-07 | 1.89956E-05 | -5.53631E-05 |
| 44.200 | 5.99826E 00 | 2.64117E-05 | -6.58464E-07 | 1.35251E-05 | -5.40506E-05 |
| 44.300 | 5.99827E 00 | 2.63571E-05 | -4.33620E-07 | 8.18439E-06 | -5.27692E-05 |
| 44.400 | 5.99827E 00 | 2.63248E-05 | -2.14107E-07 | 2.97027E-06 | -5.15182E-05 |

Pr = 0.0300

| $\eta$ | F           | F'          | F''          | $\phi$      | $\phi'$      |
|--------|-------------|-------------|--------------|-------------|--------------|
| 0.     | 0.          | 0.          | 2.46499E 00  | 4.19766E 00 | -1.00000E 00 |
| 0.100  | 1.16305E-02 | 2.25727E-01 | 2.05217E 00  | 4.09771E 00 | -9.98511E-01 |
| 0.200  | 4.38002E-02 | 4.11134E-01 | 1.66030E 00  | 3.99805E 00 | -9.94297E-01 |
| 0.300  | 9.25981E-02 | 5.58790E-01 | 1.29847E 00  | 3.89893E 00 | -9.87724E-01 |
| 0.400  | 1.54412E-01 | 6.72063E-01 | 9.73593E-01  | 3.80057E 00 | -9.79142E-01 |
| 0.500  | 2.25996E-01 | 7.54895E-01 | 6.90201E-01  | 3.70316E 00 | -9.68872E-01 |
| 0.600  | 3.04518E-01 | 8.11558E-01 | 4.50378E-01  | 3.60684E 00 | -9.57204E-01 |
| 0.700  | 3.87581E-01 | 8.46414E-01 | 2.53807E-01  | 3.51175E 00 | -9.44392E-01 |
| 0.800  | 4.73215E-01 | 8.63683E-01 | 9.80625E-02  | 3.41800E 00 | -9.30651E-01 |
| 0.900  | 5.59861E-01 | 8.67254E-01 | -2.09165E-02 | 3.32565E 00 | -9.16165E-01 |
| 1.000  | 6.46324E-01 | 8.60555E-01 | -1.08236E-01 | 3.23478E 00 | -9.01081E-01 |
| 1.100  | 7.31727E-01 | 8.46477E-01 | -1.69440E-01 | 3.14545E 00 | -8.85521E-01 |
| 1.200  | 8.15452E-01 | 8.27355E-01 | -2.09982E-01 | 3.05769E 00 | -8.69584E-01 |
| 1.300  | 8.97091E-01 | 8.05003E-01 | -2.34829E-01 | 2.97154E 00 | -8.53348E-01 |
| 1.400  | 9.76390E-01 | 7.80770E-01 | -2.48223E-01 | 2.88703E 00 | -8.36878E-01 |
| 1.500  | 1.05321E 00 | 7.55625E-01 | -2.53589E-01 | 2.80417E 00 | -8.20229E-01 |
| 1.600  | 1.12751E 00 | 7.30231E-01 | -2.53561E-01 | 2.72299E 00 | -8.03446E-01 |
| 1.700  | 1.19927E 00 | 7.05027E-01 | -2.50069E-01 | 2.64349E 00 | -7.86569E-01 |
| 1.800  | 1.26853E 00 | 6.80287E-01 | -2.44471E-01 | 2.56568E 00 | -7.69635E-01 |
| 1.900  | 1.33535E 00 | 6.56173E-01 | -2.37680E-01 | 2.48956E 00 | -7.52676E-01 |
| 2.000  | 1.39979E 00 | 6.32771E-01 | -2.30292E-01 | 2.41514E 00 | -7.35721E-01 |
| 2.100  | 1.46192E 00 | 6.10122E-01 | -2.22676E-01 | 2.34241E 00 | -7.18799E-01 |
| 2.200  | 1.52184E 00 | 5.88236E-01 | -2.15053E-01 | 2.27138E 00 | -7.01933E-01 |
| 2.300  | 1.57960E 00 | 5.67107E-01 | -2.07549E-01 | 2.20203E 00 | -6.85153E-01 |
| 2.400  | 1.63528E 00 | 5.46720E-01 | -2.00232E-01 | 2.13434E 00 | -6.68475E-01 |
| 2.500  | 1.68897E 00 | 5.27053E-01 | -1.93135E-01 | 2.06833E 00 | -6.51923E-01 |
| 2.600  | 1.74072E 00 | 5.08085E-01 | -1.86270E-01 | 2.00396E 00 | -6.35515E-01 |
| 2.700  | 1.79060E 00 | 4.89792E-01 | -1.79640E-01 | 1.94122E 00 | -6.19269E-01 |
| 2.800  | 1.83870E 00 | 4.72149E-01 | -1.73242E-01 | 1.88010E 00 | -6.03202E-01 |
| 2.900  | 1.88506E 00 | 4.55136E-01 | -1.67070E-01 | 1.82057E 00 | -5.87329E-01 |
| 3.000  | 1.92974E 00 | 4.38728E-01 | -1.61116E-01 | 1.76262E 00 | -5.71663E-01 |
| 3.100  | 1.97282E 00 | 4.22905E-01 | -1.55373E-01 | 1.70623E 00 | -5.56217E-01 |
| 3.200  | 2.01434E 00 | 4.07647E-01 | -1.49832E-01 | 1.65137E 00 | -5.41002E-01 |
| 3.300  | 2.05437E 00 | 3.92932E-01 | -1.44488E-01 | 1.59802E 00 | -5.26027E-01 |
| 3.400  | 2.09295E 00 | 3.78743E-01 | -1.39332E-01 | 1.54616E 00 | -5.11301E-01 |
| 3.500  | 2.13013E 00 | 3.65060E-01 | -1.34358E-01 | 1.49575E 00 | -4.96833E-01 |
| 3.600  | 2.16598E 00 | 3.51866E-01 | -1.29559E-01 | 1.44667E 00 | -4.82628E-01 |
| 3.700  | 2.20052E 00 | 3.39143E-01 | -1.24929E-01 | 1.39922E 00 | -4.68691E-01 |
| 3.800  | 2.23382E 00 | 3.26874E-01 | -1.20463E-01 | 1.35304E 00 | -4.55029E-01 |
| 3.900  | 2.26591E 00 | 3.15045E-01 | -1.16153E-01 | 1.30820E 00 | -4.41644E-01 |
| 4.000  | 2.29664E 00 | 3.03639E-01 | -1.11995E-01 | 1.26470E 00 | -4.28540E-01 |
| 4.100  | 2.32665E 00 | 2.92641E-01 | -1.07984E-01 | 1.22249E 00 | -4.15718E-01 |
| 4.200  | 2.35538E 00 | 2.82037E-01 | -1.04114E-01 | 1.18154E 00 | -4.0318CE-01 |
| 4.300  | 2.38307E 00 | 2.71813E-01 | -1.00381E-01 | 1.14184E 00 | -3.90927E-01 |
| 4.400  | 2.40976E 00 | 2.61956E-01 | -9.67793E-01 | 1.10335E 00 | -3.78959E-01 |
| 4.500  | 2.43548E 00 | 2.52453E-01 | -9.33048E-02 | 1.06604E 00 | -3.67276E-01 |
| 4.600  | 2.46026E 00 | 2.43291E-01 | -8.99530E-02 | 1.02988E 00 | -3.55876E-01 |
| 4.700  | 2.48415E 00 | 2.34459E-01 | -8.67196E-02 | 9.94855E-01 | -3.44759E-01 |
| 4.800  | 2.50716E 00 | 2.25944E-01 | -8.36007E-02 | 9.60923E-01 | -3.33922E-01 |
| 4.900  | 2.52934E 00 | 2.17735E-01 | -8.05922E-02 | 9.28062E-01 | -3.23362E-01 |
| 5.000  | 2.55072E 00 | 2.09822E-01 | -7.76903E-02 | 8.96242E-01 | -3.13079E-01 |
| 5.100  | 2.57132E 00 | 2.02193E-01 | -7.48913E-02 | 8.65437E-01 | -3.03067E-01 |
| 5.200  | 2.59117E 00 | 1.94840E-01 | -7.21916E-02 | 8.35619E-01 | -2.93235E-01 |
| 5.300  | 2.61030E 00 | 1.87752E-01 | -6.95879E-02 | 8.06763E-01 | -2.83848E-01 |
| 5.400  | 2.62873E 00 | 1.80919E-01 | -6.70767E-02 | 7.78841E-01 | -2.74632E-01 |
| 5.500  | 2.64649E 00 | 1.74334E-01 | -6.46549E-02 | 7.51828E-01 | -2.66675E-01 |
| 5.600  | 2.66360E 00 | 1.67986E-01 | -6.23193E-02 | 7.25698E-01 | -2.56970E-01 |
| 5.700  | 2.68009E 00 | 1.61867E-01 | -6.00669E-02 | 7.00426E-01 | -2.48514E-01 |
| 5.800  | 2.69598E 00 | 1.55970E-01 | -5.78949E-02 | 6.75987E-01 | -2.40303E-01 |
| 5.900  | 2.71129E 00 | 1.50285E-01 | -5.58005E-02 | 6.52357E-01 | -2.32332E-01 |
| 6.000  | 2.72605E 00 | 1.44807E-01 | -5.37808E-02 | 6.29512E-01 | -2.24959E-01 |
| 6.100  | 2.74026E 00 | 1.39527E-01 | -5.18334E-02 | 6.07430E-01 | -2.17089E-01 |
| 6.200  | 2.75739E 00 | 1.34438E-01 | -4.99557E-02 | 5.86087E-01 | -2.09808E-01 |
| 6.300  | 2.77617E 00 | 1.29533E-01 | -4.81552E-02 | 5.65461E-01 | -2.02747E-01 |
| 6.400  | 2.77987E 00 | 1.24807E-01 | -4.63996E-02 | 5.45530E-01 | -1.95902E-01 |
| 6.500  | 2.79212E 00 | 1.20251E-01 | -4.47166E-02 | 5.26274E-01 | -1.89267E-01 |
| 6.600  | 2.80393E 00 | 1.15861E-01 | -4.30940E-02 | 5.07670E-01 | -1.82838E-01 |
| 6.700  | 2.81530E 00 | 1.11631E-01 | -4.15297E-02 | 4.89700E-01 | -1.76609E-01 |
| 6.800  | 2.82626E 00 | 1.07554E-01 | -4.00216E-02 | 4.72342E-01 | -1.70575E-01 |

Pr = 0.0300 (Contd.)

| $\eta$ | F           | F'           | F''          | $\phi$      | $\phi'$      |
|--------|-------------|--------------|--------------|-------------|--------------|
| 6.900  | 2.83682E 00 | 1.03625E-01  | -3.85677E-02 | 4.55578E-01 | -1.64732E-01 |
| 7.000  | 2.84699E 00 | 9.98383E-02  | -3.71662E-02 | 4.39389E-01 | -1.59075E-01 |
| 7.100  | 2.85679E 00 | 9.61897E-02  | -3.58151E-02 | 4.23757E-01 | -1.53598E-01 |
| 7.200  | 2.86623E 00 | 9.26737E-02  | -3.45127E-02 | 4.08664E-01 | -1.48297E-01 |
| 7.300  | 2.87533E 00 | 8.92856E-02  | -3.32572E-02 | 3.94092E-01 | -1.43168E-01 |
| 7.400  | 2.88409E 00 | 8.60207E-02  | -3.20471E-02 | 3.80025E-01 | -1.38205E-01 |
| 7.500  | 2.89253E 00 | 8.28747E-02  | -3.08806E-02 | 3.66446E-01 | -1.33404E-01 |
| 7.600  | 2.90067E 00 | 7.98432E-02  | -2.97562E-02 | 3.53339E-01 | -1.28766E-01 |
| 7.700  | 2.90851E 00 | 7.69221E-02  | -2.86725E-02 | 3.40688E-01 | -1.24269E-01 |
| 7.800  | 2.91606E 00 | 7.41074E-02  | -2.76727E-02 | 3.28480E-01 | -1.19927E-01 |
| 7.900  | 2.92333E 00 | 7.13953E-02  | -2.66211E-02 | 3.16698E-01 | -1.15729E-01 |
| 8.000  | 2.93034E 00 | 6.87820E-02  | -2.56508E-02 | 3.05329E-01 | -1.11671E-01 |
| 8.100  | 2.93709E 00 | 6.62639E-02  | -2.47156E-02 | 2.94360E-01 | -1.07748E-01 |
| 8.200  | 2.94360E 00 | 6.38377E-02  | -2.38142E-02 | 2.83775E-01 | -1.03957E-01 |
| 8.300  | 2.94986E 00 | 6.15000E-02  | -2.29455E-02 | 2.73564E-01 | -1.00294E-01 |
| 8.400  | 2.95590E 00 | 5.92476E-02  | -2.21083E-02 | 2.63712E-01 | -9.67534E-02 |
| 8.500  | 2.96171E 00 | 5.70773E-02  | -2.13015E-02 | 2.54209E-01 | -9.33348E-02 |
| 8.600  | 2.96732E 00 | 5.49863E-02  | -2.05239E-02 | 2.45042E-01 | -9.00317E-02 |
| 8.700  | 2.97271E 00 | 5.29716E-02  | -1.97746E-02 | 2.36199E-01 | -8.68411E-02 |
| 8.800  | 2.97791E 00 | 5.10305E-02  | -1.90524E-02 | 2.27670E-01 | -8.37597E-02 |
| 8.900  | 2.98292E 00 | 4.91602E-02  | -1.83565E-02 | 2.19443E-01 | -8.07840E-02 |
| 9.000  | 2.98775E 00 | 4.73938E-02  | -1.76859E-02 | 2.11510E-01 | -7.79106E-02 |
| 9.100  | 2.99240E 00 | 4.56222E-02  | -1.70396E-02 | 2.03858E-01 | -7.51362E-02 |
| 9.200  | 2.99687E 00 | 4.39496E-02  | -1.64169E-02 | 1.96479E-01 | -7.24576E-02 |
| 9.300  | 3.00119E 00 | 4.23381E-02  | -1.58168E-02 | 1.89363E-01 | -6.98718E-02 |
| 9.400  | 3.00534E 00 | 4.07855E-02  | -1.52385E-02 | 1.82502E-01 | -6.73758E-02 |
| 9.500  | 3.00935E 00 | 3.92897E-02  | -1.46813E-02 | 1.75885E-01 | -6.49665E-02 |
| 9.600  | 3.01320E 00 | 3.78486E-02  | -1.41443E-02 | 1.69506E-01 | -6.26412E-02 |
| 9.700  | 3.01692E 00 | 3.64602E-02  | -1.36269E-02 | 1.63354E-01 | -6.03971E-02 |
| 9.800  | 3.02050E 00 | 3.51226E-02  | -1.31283E-02 | 1.57424E-01 | -5.82315E-02 |
| 9.900  | 3.02395E 00 | 3.38339E-02  | -1.26479E-02 | 1.51706E-01 | -5.61417E-02 |
| 10.000 | 3.02727E 00 | 3.25924E-02  | -1.21851E-02 | 1.46193E-01 | -5.41254E-02 |
| 10.100 | 3.03047E 00 | 3.13964E-02  | -1.17390E-02 | 1.40878E-01 | -5.21799E-02 |
| 10.200 | 3.03355E 00 | 3.02441E-02  | -1.13093E-02 | 1.35755E-01 | -5.03209E-02 |
| 10.300 | 3.03652E 00 | 2.91340E-02  | -1.08952E-02 | 1.30815E-01 | -4.84922E-02 |
| 10.400 | 3.03937E 00 | 2.80645E-02  | -1.04962E-02 | 1.26054E-01 | -4.67454E-02 |
| 10.500 | 3.04213E 00 | 2.70343E-02  | -1.01118E-02 | 1.21464E-01 | -4.50604E-02 |
| 10.600 | 3.04478E 00 | 2.60417E-02  | -9.74135E-03 | 1.17040E-01 | -4.34551E-02 |
| 10.700 | 3.04734E 00 | 2.50855E-02  | -9.38445E-03 | 1.12775E-01 | -4.18674E-02 |
| 10.800 | 3.04980E 00 | 2.41644E-02  | -9.04059E-03 | 1.08665E-01 | -4.03555E-02 |
| 10.900 | 3.05217E 00 | 2.32770E-02  | -8.70928E-03 | 1.04702E-01 | -3.88972E-02 |
| 11.000 | 3.05446E 00 | 2.24221E-02  | -8.39006E-03 | 1.00883E-01 | -3.74909E-02 |
| 11.100 | 3.05666E 00 | 2.15966E-02  | -8.08250E-03 | 9.72026E-02 | -3.61347E-02 |
| 11.200 | 3.05878E 00 | 2.08053E-02  | -7.78618E-03 | 9.36549E-02 | -3.48269E-02 |
| 11.300 | 3.06082E 00 | 2.00410E-02  | -7.50068E-03 | 9.02356E-02 | -3.35568E-02 |
| 11.400 | 3.06279E 00 | 1.93048E-02  | -7.22561E-03 | 8.69402E-02 | -3.23498E-02 |
| 11.500 | 3.06468E 00 | 1.85955E-02  | -6.96059E-03 | 8.37642E-02 | -3.11773E-02 |
| 11.600 | 3.06651E 00 | 1.79123E-02  | -6.70526E-03 | 8.07034E-02 | -3.00468E-02 |
| 11.700 | 3.06827E 00 | 1.725542E-02 | -6.45926E-03 | 7.77535E-02 | -2.89568E-02 |
| 11.800 | 3.06996E 00 | 1.66202E-02  | -6.22225E-03 | 7.49107E-02 | -2.79059E-02 |
| 11.900 | 3.07159E 00 | 1.60094E-02  | -5.99391E-03 | 7.21711E-02 | -2.68928E-02 |
| 12.000 | 3.07316E 00 | 1.54211E-02  | -5.77392E-03 | 6.95309E-02 | -2.59161E-02 |
| 12.100 | 3.07468E 00 | 1.48544E-02  | -5.56197E-03 | 6.69867E-02 | -2.49745E-02 |
| 12.200 | 3.07613E 00 | 1.43085E-02  | -5.35777E-03 | 6.45349E-02 | -2.40668E-02 |
| 12.300 | 3.07754E 00 | 1.37826E-02  | -5.16104E-03 | 6.21722E-02 | -2.31918E-02 |
| 12.400 | 3.07889E 00 | 1.32760E-02  | -4.97151E-03 | 5.98955E-02 | -2.23483E-02 |
| 12.500 | 3.08019E 00 | 1.27881E-02  | -4.78891E-03 | 5.77016E-02 | -2.15352E-02 |
| 12.600 | 3.08145E 00 | 1.23180E-02  | -4.61300E-03 | 5.55875E-02 | -2.07515E-02 |
| 12.700 | 3.08266E 00 | 1.18652E-02  | -4.44352E-03 | 5.35503E-02 | -1.99961E-02 |
| 12.800 | 3.08382E 00 | 1.14291E-02  | -4.28024E-03 | 5.15873E-02 | -1.92680E-02 |
| 12.900 | 3.08494E 00 | 1.10090E-02  | -4.12294E-03 | 4.96958E-02 | -1.85662E-02 |
| 13.000 | 3.08602E 00 | 1.06043E-02  | -3.97113E-03 | 4.78732E-02 | -1.78898E-02 |
| 13.100 | 3.08707E 00 | 1.02145E-02  | -3.82539E-03 | 4.61171E-02 | -1.72379E-02 |
| 13.200 | 3.08807E 00 | 9.83907E-03  | -3.68474E-03 | 4.44249E-02 | -1.66095E-02 |
| 13.300 | 3.08930E 00 | 9.47741E-03  | -3.54924E-03 | 4.27944E-02 | -1.60040E-02 |
| 13.400 | 3.08996E 00 | 9.12905E-03  | -3.41869E-03 | 4.12234E-02 | -1.54204E-02 |
| 13.500 | 3.09086E 00 | 8.79351E-03  | -3.29293E-03 | 3.97096E-02 | -1.48579E-02 |
| 13.600 | 3.09172E 00 | 8.47032E-03  | -3.17177E-03 | 3.82511E-02 | -1.43158E-02 |
| 13.700 | 3.09255E 00 | 8.15901E-03  | -3.05505E-03 | 3.68458E-02 | -1.37935E-02 |

**Pr = 0.0300 (Contd.)**

| $\eta$ | F           | F'          | F''          | $\phi$      | $\phi'$      |
|--------|-------------|-------------|--------------|-------------|--------------|
| 13.800 | 3.09336E 00 | 7.85916E-03 | -2.94260E-03 | 3.54918E-02 | -1.32900E-02 |
| 13.900 | 3.09413E 00 | 7.57035E-03 | -2.83428E-03 | 3.41872E-02 | -1.28049E-02 |
| 14.000 | 3.09487E 00 | 7.29217E-03 | -2.72992E-03 | 3.29302E-02 | -1.23374E-02 |
| 14.100 | 3.09559E 00 | 7.02424E-03 | -2.62938E-03 | 3.17191E-02 | -1.18869E-02 |
| 14.200 | 3.09627E 00 | 6.76618E-03 | -2.53252E-03 | 3.05523E-02 | -1.14527E-02 |
| 14.300 | 3.09694E 00 | 6.51762E-03 | -2.43921E-03 | 2.94281E-02 | -1.10344E-02 |
| 14.400 | 3.09758E 00 | 6.27822E-03 | -2.34932E-03 | 2.83449E-02 | -1.06312E-02 |
| 14.500 | 3.09819E 00 | 6.04764E-03 | -2.26273E-03 | 2.73013L-02 | -1.02426E-02 |
| 14.600 | 3.09879E 00 | 5.82557E-03 | -2.17930E-03 | 2.62959E-02 | -9.86846E-03 |
| 14.700 | 3.09936E 00 | 5.61168E-03 | -2.09894E-03 | 2.53272E-02 | -9.50777E-03 |
| 14.800 | 3.09991E 00 | 5.40568E-03 | -2.02152E-03 | 2.43939E-02 | -9.16022E-03 |
| 14.900 | 3.10044E 00 | 5.20728E-03 | -1.94693E-03 | 2.34947E-02 | -8.82533E-03 |
| 15.000 | 3.10095E 00 | 5.01621E-03 | -1.87508E-03 | 2.26284E-02 | -8.50264E-03 |
| 15.100 | 3.10145E 00 | 4.83218E-03 | -1.80586E-03 | 2.17938E-02 | -8.19171E-03 |
| 15.200 | 3.10192E 00 | 4.65495E-03 | -1.73918E-03 | 2.09897E-02 | -7.89212E-03 |
| 15.300 | 3.10238E 00 | 4.48426E-03 | -1.67494E-03 | 2.02150E-02 | -7.63456E-03 |
| 15.400 | 3.10282E 00 | 4.31988E-03 | -1.61305E-03 | 1.94686E-02 | -7.32531E-03 |
| 15.500 | 3.10324E 00 | 4.16158E-03 | -1.55344E-03 | 1.87496E-02 | -7.05732E-03 |
| 15.600 | 3.10365E 00 | 4.00912E-03 | -1.49600E-03 | 1.80569E-02 | -6.79911E-03 |
| 15.700 | 3.10404E 00 | 3.86231E-03 | -1.44068E-03 | 1.73895E-02 | -6.55032E-03 |
| 15.800 | 3.10442E 00 | 3.72092E-03 | -1.38738E-03 | 1.67465E-02 | -6.31062E-03 |
| 15.900 | 3.10479E 00 | 3.58477E-03 | -1.33603E-03 | 1.61270E-02 | -6.07966E-03 |
| 16.000 | 3.10514E 00 | 3.45365E-03 | -1.28657E-03 | 1.55303E-02 | -5.85714E-03 |
| 16.100 | 3.10548E 00 | 3.32739E-03 | -1.23892E-03 | 1.49553E-02 | -5.66275E-03 |
| 16.200 | 3.10580E 00 | 3.20581E-03 | -1.19302E-03 | 1.44015E-02 | -5.43618E-03 |
| 16.300 | 3.10612E 00 | 3.08873E-03 | -1.14880E-03 | 1.38679E-02 | -5.23717E-03 |
| 16.400 | 3.10642E 00 | 2.97599E-03 | -1.10620E-03 | 1.33536E-02 | -5.04542E-03 |
| 16.500 | 3.10671E 00 | 2.86744E-03 | -1.06516E-03 | 1.28585E-02 | -4.86068E-03 |
| 16.600 | 3.10700E 00 | 2.76291E-03 | -1.02563E-03 | 1.23814E-02 | -4.68270E-03 |
| 16.700 | 3.10727E 00 | 2.66226E-03 | -9.87547E-04 | 1.19218E-02 | -4.51122E-03 |
| 16.800 | 3.10753E 00 | 2.56536E-03 | -9.50859E-04 | 1.14790E-02 | -4.34601E-03 |
| 16.900 | 3.10778E 00 | 2.47205E-03 | -9.15517E-04 | 1.10524E-02 | -4.18684E-03 |
| 17.000 | 3.10802E 00 | 2.38822E-03 | -8.81471E-04 | 1.06414E-02 | -4.03349E-03 |
| 17.100 | 3.10826E 00 | 2.29571E-03 | -8.48673E-04 | 1.02455E-02 | -3.88575E-03 |
| 17.200 | 3.10848E 00 | 2.21243E-03 | -8.17078E-04 | 9.86408E-03 | -3.74341E-03 |
| 17.300 | 3.10870E 00 | 2.13226E-03 | -7.86641E-04 | 9.49664E-03 | -3.60268E-03 |
| 17.400 | 3.10891E 00 | 2.05507E-03 | -7.57320E-04 | 9.14266E-03 | -3.47417E-03 |
| 17.500 | 3.10911E 00 | 1.98076E-03 | -7.29074E-04 | 8.80164E-03 | -3.34689E-03 |
| 17.600 | 3.10931E 00 | 1.90922E-03 | -7.01864E-04 | 8.47312E-03 | -3.22427E-03 |
| 17.700 | 3.10949E 00 | 1.84035E-03 | -6.75652E-04 | 8.15664E-03 | -3.10613E-03 |
| 17.800 | 3.10967E 00 | 1.77406E-03 | -6.50401E-04 | 7.85175E-03 | -2.99232E-03 |
| 17.900 | 3.10985E 00 | 1.71024E-03 | -6.26076E-04 | 7.55804E-03 | -2.88267E-03 |
| 18.000 | 3.11002E 00 | 1.64881E-03 | -6.02642E-04 | 7.27508E-03 | -2.77704E-03 |
| 18.100 | 3.11018E 00 | 1.58968E-03 | -5.80069E-04 | 7.00250E-03 | -2.67527E-03 |
| 18.200 | 3.11033E 00 | 1.53277E-03 | -5.58323E-04 | 6.73991E-03 | -2.57723E-03 |
| 18.300 | 3.11048E 00 | 1.47799E-03 | -5.37374E-04 | 6.48693E-03 | -2.48278E-03 |
| 18.400 | 3.11063E 00 | 1.42527E-03 | -5.17194E-04 | 6.24323E-03 | -2.39179E-03 |
| 18.500 | 3.11077E 00 | 1.37453E-03 | -4.97754E-04 | 6.00847E-03 | -2.30413E-03 |
| 18.600 | 3.11090E 00 | 1.32570E-03 | -4.79027E-04 | 5.78230E-03 | -2.21968E-03 |
| 18.700 | 3.11103E 00 | 1.27870E-03 | -4.60986E-04 | 5.56443E-03 | -2.13832E-03 |
| 18.800 | 3.11116E 00 | 1.23348E-03 | -4.43608E-04 | 5.35454E-03 | -2.05994E-03 |
| 18.900 | 3.11128E 00 | 1.18996E-03 | -4.26866E-04 | 5.15234E-03 | -1.98443E-03 |
| 19.000 | 3.11140E 00 | 1.14808E-03 | -4.10739E-04 | 4.95756E-03 | -1.91169E-03 |
| 19.100 | 3.11151E 00 | 1.10779E-03 | -3.95203E-04 | 4.76992E-03 | -1.84161E-03 |
| 19.200 | 3.11162E 00 | 1.06902E-03 | -3.80237E-04 | 4.58915E-03 | -1.77410E-03 |
| 19.300 | 3.11172E 00 | 1.03173E-03 | -3.65819E-04 | 4.41501E-03 | -1.70906E-03 |
| 19.400 | 3.11183E 00 | 9.95842E-04 | -3.51931E-04 | 4.24726E-03 | -1.64641E-03 |
| 19.500 | 3.11192E 00 | 9.61322E-04 | -3.38552E-04 | 4.08566E-03 | -1.58605E-03 |
| 19.600 | 3.11202E 00 | 9.28115E-04 | -3.25663E-04 | 3.92998E-03 | -1.52790E-03 |
| 19.700 | 3.11211E 00 | 8.96174E-04 | -3.13248E-04 | 3.78000E-03 | -1.47188E-03 |
| 19.800 | 3.11220E 00 | 8.65450E-04 | -3.01288E-04 | 3.63553E-03 | -1.41792E-03 |
| 19.900 | 3.11228E 00 | 8.35901E-04 | -2.89766E-04 | 3.49635E-03 | -1.36933E-03 |
| 20.000 | 3.11236E 00 | 8.07483E-04 | -2.78667E-04 | 3.36228E-03 | -1.31585E-03 |
| 20.100 | 3.11244E 00 | 7.80154E-04 | -2.67975E-04 | 3.23312E-03 | -1.26760E-03 |
| 20.200 | 3.11252E 00 | 7.53875E-04 | -2.57675E-04 | 3.10870E-03 | -1.22113E-03 |
| 20.300 | 3.11259E 00 | 7.28607E-04 | -2.47754E-04 | 2.98884E-03 | -1.17635E-03 |
| 20.400 | 3.11267E 00 | 7.04312E-04 | -2.38196E-04 | 2.87338E-03 | -1.13322E-03 |
| 20.500 | 3.11274E 00 | 6.80956E-04 | -2.28988E-04 | 2.76215E-03 | -1.09166E-03 |
| 20.600 | 3.11280E 00 | 6.58503E-04 | -2.20118E-04 | 2.65499E-03 | -1.05163E-03 |

**Pr = 0.0300 (Contd.)**

| $\eta$ | F           | F'          | F''          | $\phi$      | $\phi'$      |
|--------|-------------|-------------|--------------|-------------|--------------|
| 20.700 | 3.11287E 00 | 6.36921E-04 | -2.11574E-04 | 2.55177E-03 | -1.01307E-03 |
| 20.800 | 3.11293E 00 | 6.16178E-04 | -2.03343E-04 | 2.45233E-03 | -9.75920E-04 |
| 20.900 | 3.11299E 00 | 5.96243E-04 | -1.95141E-04 | 2.35654E-03 | -9.40132E-04 |
| 21.000 | 3.11305E 00 | 5.77086E-04 | -1.87776E-04 | 2.26426E-03 | -9.05656E-04 |
| 21.100 | 3.11311E 00 | 5.58678E-04 | -1.80418E-04 | 2.17537E-03 | -8.72444E-04 |
| 21.200 | 3.11316E 00 | 5.40993E-04 | -1.73330E-04 | 2.08973E-03 | -8.40449E-04 |
| 21.300 | 3.11321E 00 | 5.24003E-04 | -1.66502E-04 | 2.00724E-03 | -8.09628E-04 |
| 21.400 | 3.11327E 00 | 5.07684E-04 | -1.59925E-04 | 1.92777E-03 | -7.79936E-04 |
| 21.500 | 3.11332E 00 | 4.92010E-04 | -1.53588E-04 | 1.85121E-03 | -7.51333E-04 |
| 21.600 | 3.11336E 00 | 4.76959E-04 | -1.47484E-04 | 1.77747E-03 | -7.23778E-04 |
| 21.700 | 3.11341E 00 | 4.62506E-04 | -1.41604E-04 | 1.70643E-03 | -6.97234E-04 |
| 21.800 | 3.11346E 00 | 4.48631E-04 | -1.35940E-04 | 1.63779E-03 | -6.71663E-04 |
| 21.900 | 3.11350E 00 | 4.35311E-04 | -1.30484E-04 | 1.57206E-03 | -6.47029E-04 |
| 22.000 | 3.11354E 00 | 4.22527E-04 | -1.25227E-04 | 1.50855E-03 | -6.23299E-04 |
| 22.100 | 3.11359E 00 | 4.10259E-04 | -1.20164E-04 | 1.44737E-03 | -6.00438E-04 |
| 22.200 | 3.11363E 00 | 3.98488E-04 | -1.15286E-04 | 1.38844E-03 | -5.78416E-04 |
| 22.300 | 3.11367E 00 | 3.87196E-04 | -1.10587E-04 | 1.33166E-03 | -5.57202E-04 |
| 22.400 | 3.11370E 00 | 3.76365E-04 | -1.06061E-04 | 1.27697E-03 | -5.36765E-04 |
| 22.500 | 3.11374E 00 | 3.65978E-04 | -1.01701E-04 | 1.22428E-03 | -5.17078E-04 |
| 22.600 | 3.11378E 00 | 3.56020E-04 | -9.75002E-05 | 1.17353E-03 | -4.98112E-04 |
| 22.700 | 3.11381E 00 | 3.46473E-04 | -9.34538E-05 | 1.12464E-03 | -4.79842E-04 |
| 22.800 | 3.11385E 00 | 3.37324E-04 | -8.95560E-05 | 1.07754E-03 | -4.62242E-04 |
| 22.900 | 3.11388E 00 | 3.28557E-04 | -8.58011E-05 | 1.03217E-03 | -4.45288E-04 |
| 23.000 | 3.11391E 00 | 3.20159E-04 | -8.21839E-05 | 9.88462E-04 | -4.28956E-04 |
| 23.100 | 3.11394E 00 | 3.12116E-04 | -7.86995E-05 | 9.46358E-04 | -4.13221E-04 |
| 23.200 | 3.11397E 00 | 3.04415E-04 | -7.53429E-05 | 9.05799E-04 | -3.98064E-04 |
| 23.300 | 3.11400E 00 | 2.97043E-04 | -7.21094E-05 | 8.66727E-04 | -3.83463E-04 |
| 23.400 | 3.11403E 00 | 2.89989E-04 | -6.89946E-05 | 8.29088E-04 | -3.69397E-04 |
| 23.500 | 3.11406E 00 | 2.83241E-04 | -6.59940E-05 | 7.92830E-04 | -3.55847E-04 |
| 23.600 | 3.11409E 00 | 2.76787E-04 | -6.31035E-05 | 7.57902E-04 | -3.42794E-04 |
| 23.700 | 3.11412E 00 | 2.70616E-04 | -6.03191E-05 | 7.24255E-04 | -3.30220E-04 |
| 23.800 | 3.11414E 00 | 2.64719E-04 | -5.76368E-05 | 6.91843E-04 | -3.18107E-04 |
| 23.900 | 3.11417E 00 | 2.59086E-04 | -5.50529E-05 | 6.60619E-04 | -3.06439E-04 |
| 24.000 | 3.11420E 00 | 2.53706E-04 | -5.25638E-05 | 6.30541E-04 | -2.95198E-04 |
| 24.100 | 3.11422E 00 | 2.48570E-04 | -5.01661E-05 | 6.01566E-04 | -2.84369E-04 |
| 24.200 | 3.11425E 00 | 2.43670E-04 | -4.78563E-05 | 5.73654E-04 | -2.73938E-04 |
| 24.300 | 3.11427E 00 | 2.38996E-04 | -4.56312E-05 | 5.46765E-04 | -2.63889E-04 |
| 24.400 | 3.11429E 00 | 2.34541E-04 | -4.34878E-05 | 5.20864E-04 | -2.54209E-04 |
| 24.500 | 3.11432E 00 | 2.30296E-04 | -4.14230E-05 | 4.95912E-04 | -2.44883E-04 |
| 24.600 | 3.11434E 00 | 2.26253E-04 | -3.94339E-05 | 4.71876E-04 | -2.35900E-04 |
| 24.700 | 3.11436E 00 | 2.22406E-04 | -3.75179E-05 | 4.48721E-04 | -2.27247E-04 |
| 24.800 | 3.11438E 00 | 2.18748E-04 | -3.56721E-05 | 4.26416E-04 | -2.18910E-04 |
| 24.900 | 3.11441E 00 | 2.15270E-04 | -3.38941E-05 | 4.04929E-04 | -2.10880E-04 |
| 25.000 | 3.11443E 00 | 2.11967E-04 | -3.21813E-05 | 3.84230E-04 | -2.03144E-04 |
| 25.100 | 3.11445E 00 | 2.08831E-04 | -3.05313E-05 | 3.64290E-04 | -1.95691E-04 |
| 25.200 | 3.11447E 00 | 2.05858E-04 | -2.89419E-05 | 3.45083E-04 | -1.88513E-04 |
| 25.300 | 3.11449E 00 | 2.03041E-04 | -2.74108E-05 | 3.26579E-04 | -1.81597E-04 |
| 25.400 | 3.11451E 00 | 2.00374E-04 | -2.59358E-05 | 3.08755E-04 | -1.74935E-04 |
| 25.500 | 3.11453E 00 | 1.97852E-04 | -2.45150E-05 | 2.91584E-04 | -1.68517E-04 |
| 25.600 | 3.11455E 00 | 1.95649E-04 | -2.31463E-05 | 2.75043E-04 | -1.62335E-04 |
| 25.700 | 3.11457E 00 | 1.93221E-04 | -2.18278E-05 | 2.59109E-04 | -1.56380E-04 |
| 25.800 | 3.11459E 00 | 1.91102E-04 | -2.05577E-05 | 2.43760E-04 | -1.50643E-04 |
| 25.900 | 3.11461E 00 | 1.89108E-04 | -1.93342E-05 | 2.28974E-04 | -1.45116E-04 |
| 26.000 | 3.11463E 00 | 1.87234E-04 | -1.81556E-05 | 2.14730E-04 | -1.39793E-04 |
| 26.100 | 3.11464E 00 | 1.85676E-04 | -1.70203E-05 | 2.01009E-04 | -1.34664E-04 |
| 26.200 | 3.11466E 00 | 1.83829E-04 | -1.59266E-05 | 1.87791E-04 | -1.29724E-04 |
| 26.300 | 3.11468E 00 | 1.82289E-04 | -1.48730E-05 | 1.75058E-04 | -1.24965E-04 |
| 26.400 | 3.11470E 00 | 1.80853E-04 | -1.38581E-05 | 1.62792E-04 | -1.20380E-04 |
| 26.500 | 3.11472E 00 | 1.79516E-04 | -1.28804E-05 | 1.50976E-04 | -1.15964E-04 |
| 26.600 | 3.11474E 00 | 1.78275E-04 | -1.19386E-05 | 1.39594E-04 | -1.11709E-04 |
| 26.700 | 3.11475E 00 | 1.77112E-04 | -1.10313E-05 | 1.28629E-04 | -1.07611E-04 |
| 26.800 | 3.11477E 00 | 1.76068E-04 | -1.01574E-05 | 1.18067E-04 | -1.03663E-04 |
| 26.900 | 3.11479E 00 | 1.75095E-04 | -9.31546E-06 | 1.07892E-04 | -9.98956E-05 |
| 27.000 | 3.11481E 00 | 1.74204E-04 | -8.50445E-06 | 9.80904E-05 | -9.61959E-05 |
| 27.100 | 3.11482E 00 | 1.73393E-04 | -7.72320E-06 | 8.86484E-05 | -9.26666E-05 |
| 27.200 | 3.11484E 00 | 1.72658E-04 | -6.97062E-06 | 7.95528E-05 | -8.92668E-05 |
| 27.300 | 3.11486E 00 | 1.71998E-04 | -6.24565E-06 | 7.07909E-05 | -8.59917E-05 |
| 27.400 | 3.11487E 00 | 1.71408E-04 | -5.54728E-06 | 6.23505E-05 | -8.28368E-05 |
| 27.500 | 3.11489E 00 | 1.70887E-04 | -4.87453E-06 | 5.42197E-05 | -7.97976E-05 |

Pr = 0.0300 (Contd.)

| <u><math>\eta</math></u> | F           | F'          | F''          | $\phi$      | $\phi'$      |
|--------------------------|-------------|-------------|--------------|-------------|--------------|
| 27.600                   | 3.11491E 00 | 1.70433E-04 | -4.22647E-06 | 4.63872E-05 | -7.68699E-05 |
| 27.700                   | 3.11493E 00 | 1.70041E-04 | -3.60219E-06 | 3.88421E-05 | -7.40496E-05 |
| 27.800                   | 3.11494E 00 | 1.69711E-04 | -3.00082E-06 | 3.15738E-05 | -7.13328E-05 |
| 27.900                   | 3.11496E 00 | 1.69440E-04 | -2.42151E-06 | 2.45722E-05 | -6.87156E-05 |
| 28.000                   | 3.11498E 00 | 1.69226E-04 | -1.86345E-06 | 1.78275E-05 | -6.61944E-05 |

Pr = 0.1000

| $\eta$ | F           | F'          | F''          | $\phi$      | $\phi'$      |
|--------|-------------|-------------|--------------|-------------|--------------|
| 0.     | 0.          | 0.          | 1.64342E 00  | 2.75070E 00 | -1.00000E 00 |
| 0.100  | 7.76326E-03 | 1.50777E-01 | 1.37421E 00  | 2.65077E 00 | -9.97811E-01 |
| 0.200  | 2.92811E-02 | 2.75339E-01 | 1.11987E 00  | 2.55127E 00 | -9.91529E-01 |
| 0.300  | 6.20133E-02 | 3.75380E-01 | 8.84397E-01  | 2.45259E 00 | -9.81580E-01 |
| 0.400  | 1.03608E-01 | 4.52951E-01 | 6.70880E-01  | 2.35507E 00 | -9.68385E-01 |
| 0.500  | 1.51931E-01 | 5.10359E-01 | 4.81402E-01  | 2.25901E 00 | -9.52354E-01 |
| 0.600  | 2.05090E-01 | 5.50070E-01 | 3.17036E-01  | 2.16468E 00 | -9.33880E-01 |
| 0.700  | 2.61439E-01 | 5.74607E-01 | 1.77865E-01  | 2.07230E 00 | -9.13337E-01 |
| 0.800  | 3.19588E-01 | 5.86456E-01 | 6.30655E-02  | 1.98207E 00 | -8.91074E-01 |
| 0.900  | 3.78387E-01 | 5.87980E-01 | -2.48954E-02 | 1.89413E 00 | -8.67409E-01 |
| 1.000  | 4.36913E-01 | 5.81353E-01 | -1.00363E-01 | 1.80862E 00 | -8.42634E-01 |
| 1.100  | 4.94450E-01 | 5.68510E-01 | -1.53703E-01 | 1.72563E 00 | -8.17010E-01 |
| 1.200  | 5.50464E-01 | 5.51124E-01 | -1.91679E-01 | 1.64524E 00 | -7.90769E-01 |
| 1.300  | 6.04571E-01 | 5.30597E-01 | -2.16976E-01 | 1.56749E 00 | -7.64116E-01 |
| 1.400  | 6.56517E-01 | 5.08067E-01 | -2.32119E-01 | 1.49242E 00 | -7.37234E-01 |
| 1.500  | 7.06148E-01 | 4.84435E-01 | -2.39374E-01 | 1.42005E 00 | -7.10280E-01 |
| 1.600  | 7.53390E-01 | 4.60390E-01 | -2.40691E-01 | 1.35037E 00 | -6.83394E-01 |
| 1.700  | 7.98229E-01 | 4.36441E-01 | -2.37690E-01 | 1.28336E 00 | -6.56695E-01 |
| 1.800  | 8.40694E-01 | 4.12952E-01 | -2.31674E-01 | 1.21902E 00 | -6.30288E-01 |
| 1.900  | 8.80843E-01 | 3.90172E-01 | -2.23657E-01 | 1.15729E 00 | -6.04264E-01 |
| 2.000  | 9.18757E-01 | 3.68262E-01 | -2.14406E-01 | 1.09815E 00 | -5.78700E-01 |
| 2.100  | 9.54528E-01 | 3.47313E-01 | -2.04486E-01 | 1.04154E 00 | -5.53661E-01 |
| 2.200  | 9.88253E-01 | 3.27373E-01 | -1.94299E-01 | 9.87398E-01 | -5.29203E-01 |
| 2.300  | 1.02004E 00 | 3.08453E-01 | -1.84124E-01 | 9.35675E-01 | -5.05371E-01 |
| 2.400  | 1.04998E 00 | 2.90542E-01 | -1.74147E-01 | 8.86302E-01 | -4.82202E-01 |
| 2.500  | 1.07818E 00 | 2.73613E-01 | -1.64486E-01 | 8.39211E-01 | -4.59725E-01 |
| 2.600  | 1.10473E 00 | 2.57632E-01 | -1.55221E-01 | 7.94333E-01 | -4.37963E-01 |
| 2.700  | 1.12973E 00 | 2.42557E-01 | -1.46362E-01 | 7.51594E-01 | -4.16930E-01 |
| 2.800  | 1.15327E 00 | 2.28345E-01 | -1.37952E-01 | 7.10922E-01 | -3.96638E-01 |
| 2.900  | 1.17543E 00 | 2.14951E-01 | -1.29983E-01 | 6.72242E-01 | -3.77709E-01 |
| 3.000  | 1.19629E 00 | 2.02334E-01 | -1.22446E-01 | 6.35479E-01 | -3.58288E-01 |
| 3.100  | 1.21592E 00 | 1.90448E-01 | -1.15326E-01 | 6.00560E-01 | -3.40227E-01 |
| 3.200  | 1.23440E 00 | 1.79255E-01 | -1.08607E-01 | 5.67409E-01 | -3.22902E-01 |
| 3.300  | 1.25179E 00 | 1.68714E-01 | -1.02027E-01 | 5.35955E-01 | -3.06302E-01 |
| 3.400  | 1.26816E 00 | 1.58789E-01 | -9.62960E-02 | 5.06125E-01 | -2.90415E-01 |
| 3.500  | 1.28357E 00 | 1.49444E-01 | -9.06653E-02 | 4.77849E-01 | -2.75226E-01 |
| 3.600  | 1.29807E 00 | 1.40645E-01 | -8.53594E-02 | 4.51057E-01 | -2.60719E-01 |
| 3.700  | 1.31172E 00 | 1.32361E-01 | -8.03602E-02 | 4.25683E-01 | -2.46877E-01 |
| 3.800  | 1.32456E 00 | 1.24563E-01 | -7.56507E-02 | 4.01660E-01 | -2.33679E-01 |
| 3.900  | 1.33665E 00 | 1.17222E-01 | -7.12143E-02 | 3.78926E-01 | -2.21107E-01 |
| 4.000  | 1.34802E 00 | 1.10312E-01 | -6.70356E-02 | 3.57419E-01 | -2.09141E-01 |
| 4.100  | 1.35872E 00 | 1.03807E-01 | -6.30999E-02 | 3.37079E-01 | -1.97758E-01 |
| 4.200  | 1.36879E 00 | 9.76842E-02 | -5.93932E-02 | 3.17848E-01 | -1.86938E-01 |
| 4.300  | 1.37827E 00 | 9.19212E-02 | -5.59024E-02 | 2.99673E-01 | -1.76659E-01 |
| 4.400  | 1.38719E 00 | 8.64969E-02 | -5.26152E-02 | 2.82499E-01 | -1.66901E-01 |
| 4.500  | 1.39558E 00 | 8.13918E-02 | -4.95197E-02 | 2.66276E-01 | -1.57642E-01 |
| 4.600  | 1.40348E 00 | 7.65870E-02 | -4.66051E-02 | 2.50955E-01 | -1.48861E-01 |
| 4.700  | 1.41091E 00 | 7.20651E-02 | -4.38607E-02 | 2.36489E-01 | -1.40537E-01 |
| 4.800  | 1.41790E 00 | 6.78095E-02 | -4.12769E-02 | 2.22833E-01 | -1.32650E-01 |
| 4.900  | 1.42448E 00 | 6.38046E-02 | -3.88443E-02 | 2.09945E-01 | -1.25181E-01 |
| 5.000  | 1.43067E 00 | 6.00359E-02 | -3.65542E-02 | 1.97783E-01 | -1.18111E-01 |
| 5.100  | 1.43649E 00 | 5.64893E-02 | -3.43982E-02 | 1.86310E-01 | -1.11420E-01 |
| 5.200  | 1.44197E 00 | 5.31520E-02 | -3.23688E-02 | 1.75487E-01 | -1.05090E-01 |
| 5.300  | 1.44713E 00 | 5.00116E-02 | -3.04584E-02 | 1.65280E-01 | -9.91044E-02 |
| 5.400  | 1.45198E 00 | 4.70566E-02 | -2.86602E-02 | 1.55656E-01 | -9.34458E-02 |
| 5.500  | 1.45655E 00 | 4.42761E-02 | -2.69676E-02 | 1.46581E-01 | -8.80980E-02 |
| 5.600  | 1.46084E 00 | 4.16598E-02 | -2.53745E-02 | 1.38026E-01 | -8.30454E-02 |
| 5.700  | 1.46448E 00 | 3.91980E-02 | -2.38751E-02 | 1.29963E-01 | -7.82728E-02 |
| 5.800  | 1.46869E 00 | 3.68818E-02 | -2.24640E-02 | 1.22363E-01 | -7.37660E-02 |
| 5.900  | 1.47226E 00 | 3.47025E-02 | -2.11359E-02 | 1.15201E-01 | -6.95109E-02 |
| 6.000  | 1.47563E 00 | 3.26520E-02 | -1.98860E-02 | 1.08453E-01 | -6.54944E-02 |
| 6.100  | 1.47880E 00 | 3.07228E-02 | -1.87097E-02 | 1.02095E-01 | -6.17043E-02 |
| 6.200  | 1.48178E 00 | 2.89078E-02 | -1.76028E-02 | 9.61046E-02 | -5.81281E-02 |
| 6.300  | 1.48458E 00 | 2.72001E-02 | -1.65612E-02 | 9.04622E-02 | -5.47544E-02 |
| 6.400  | 1.48722E 00 | 2.55935E-02 | -1.55810E-02 | 8.51474E-02 | -5.15723E-02 |
| 6.500  | 1.48971E 00 | 2.40820E-02 | -1.46586E-02 | 8.01416E-02 | -4.85715E-02 |
| 6.600  | 1.49204E 00 | 2.26599E-02 | -1.37906E-02 | 7.54274E-02 | -4.57420E-02 |
| 6.700  | 1.49424E 00 | 2.13221E-02 | -1.29740E-02 | 7.09879E-02 | -4.30743E-02 |
| 6.800  | 1.49631E 00 | 2.00635E-02 | -1.22055E-02 | 6.68074E-02 | -4.05597E-02 |

Pr = 0.1000 (Contd.)

| $\eta$ | F           | F'          | F''          | $\phi$      | $\psi'$      |
|--------|-------------|-------------|--------------|-------------|--------------|
| 6.900  | 1.49826E 00 | 1.88795E-02 | -1.14824E-02 | 6.28711E-02 | -3.81896E-02 |
| 7.000  | 1.50009E 00 | 1.77656E-02 | -1.08021E-02 | 5.91649E-02 | -3.59559E-02 |
| 7.100  | 1.50181E 00 | 1.67178E-02 | -1.01620E-02 | 5.56756E-02 | -3.38511E-02 |
| 7.200  | 1.50343E 00 | 1.57320E-02 | -9.55968E-03 | 5.23906E-02 | -3.18679E-02 |
| 7.300  | 1.50496E 00 | 1.48046E-02 | -8.99301E-03 | 4.92982E-02 | -2.99995E-02 |
| 7.400  | 1.50640E 00 | 1.39323E-02 | -8.45985E-03 | 4.63871E-02 | -2.82394E-02 |
| 7.500  | 1.50775E 00 | 1.31116E-02 | -7.95823E-03 | 4.36469E-02 | -2.65814E-02 |
| 7.600  | 1.50902E 00 | 1.23396E-02 | -7.48629E-03 | 4.10676E-02 | -2.50198E-02 |
| 7.700  | 1.51022E 00 | 1.16134E-02 | -7.04227E-03 | 3.86399E-02 | -2.35941E-02 |
| 7.800  | 1.51134E 00 | 1.09303E-02 | -6.62453E-03 | 3.63550E-02 | -2.21640E-02 |
| 7.900  | 1.51240E 00 | 1.02877E-02 | -6.23152E-03 | 3.42044E-02 | -2.08598E-02 |
| 8.000  | 1.51340E 00 | 9.68322E-03 | -5.86178E-03 | 3.21805E-02 | -1.96316E-02 |
| 8.100  | 1.51434E 00 | 9.11461E-03 | -5.51393E-03 | 3.02757E-02 | -1.84753E-02 |
| 8.200  | 1.51523E 00 | 8.57974E-03 | -5.18668E-03 | 2.84832E-02 | -1.73865E-02 |
| 8.300  | 1.51606E 00 | 8.07663E-03 | -4.87881E-03 | 2.67963E-02 | -1.63615E-02 |
| 8.400  | 1.51684E 00 | 7.60337E-03 | -4.58918E-03 | 2.52089E-02 | -1.53966E-02 |
| 8.500  | 1.51758E 00 | 7.15822E-03 | -4.31670E-03 | 2.37151E-02 | -1.44883E-02 |
| 8.600  | 1.51827E 00 | 6.73950E-03 | -4.06037E-03 | 2.23094E-02 | -1.36332E-02 |
| 8.700  | 1.51893E 00 | 6.34564E-03 | -3.81922E-03 | 2.09868E-02 | -1.28283E-02 |
| 8.800  | 1.51954E 00 | 5.97518E-03 | -3.59237E-03 | 1.97422E-02 | -1.20708E-02 |
| 8.900  | 1.52012E 00 | 5.62672E-03 | -3.37895E-03 | 1.85711E-02 | -1.13577E-02 |
| 9.000  | 1.52067E 00 | 5.29896E-03 | -3.17819E-03 | 1.74692E-02 | -1.06866E-02 |
| 9.100  | 1.52119E 00 | 4.99068E-03 | -2.98933E-03 | 1.64325E-02 | -1.00550E-02 |
| 9.200  | 1.52167E 00 | 4.70073E-03 | -2.81166E-03 | 1.54570E-02 | -9.46059E-03 |
| 9.300  | 1.52213E 00 | 4.42800E-03 | -2.64452E-03 | 1.45392E-02 | -8.90118E-03 |
| 9.400  | 1.52256E 00 | 4.17149E-03 | -2.48730E-03 | 1.36757E-02 | -8.37474E-03 |
| 9.500  | 1.52296E 00 | 3.93023E-03 | -2.33939E-03 | 1.28632E-02 | -7.87933E-03 |
| 9.600  | 1.52334E 00 | 3.70332E-03 | -2.20025E-03 | 1.20988E-02 | -7.41315E-03 |
| 9.700  | 1.52370E 00 | 3.48991E-03 | -2.06937E-03 | 1.13797E-02 | -6.97446E-03 |
| 9.800  | 1.52404E 00 | 3.28919E-03 | -1.94642E-03 | 1.07031E-02 | -6.56167E-03 |
| 9.900  | 1.52436E 00 | 3.10041E-03 | -1.83042E-03 | 1.00665E-02 | -6.17256E-03 |
| 10.000 | 1.52466E 00 | 2.92288E-03 | -1.72147E-03 | 9.46766E-03 | -5.80776E-03 |
| 10.100 | 1.52495E 00 | 2.75591E-03 | -1.61897E-03 | 8.90425E-03 | -5.46387E-03 |
| 10.200 | 1.52521E 00 | 2.59888E-03 | -1.52256E-03 | 8.37421E-03 | -5.14029E-03 |
| 10.300 | 1.52547E 00 | 2.45120E-03 | -1.43186E-03 | 7.87556E-03 | -4.83584E-03 |
| 10.400 | 1.52570E 00 | 2.31233E-03 | -1.34655E-03 | 7.40644E-03 | -4.54939E-03 |
| 10.500 | 1.52593E 00 | 2.18172E-03 | -1.26629E-03 | 6.96511E-03 | -4.27988E-03 |
| 10.600 | 1.52614E 00 | 2.05891E-03 | -1.19080E-03 | 6.54993E-03 | -4.02630E-03 |
| 10.700 | 1.52634E 00 | 1.94342E-03 | -1.11978E-03 | 6.15935E-03 | -3.78773E-03 |
| 10.800 | 1.52653E 00 | 1.83481E-03 | -1.05298E-03 | 5.79192E-03 | -3.56227E-03 |
| 10.900 | 1.52671E 00 | 1.73269E-03 | -9.90142E-04 | 5.44626E-03 | -3.35209E-03 |
| 11.000 | 1.52688E 00 | 1.63666E-03 | -9.31031E-04 | 5.12108E-03 | -3.15341E-03 |
| 11.100 | 1.52703E 00 | 1.54636E-03 | -8.75426E-04 | 4.81518E-03 | -2.96649E-03 |
| 11.200 | 1.52718E 00 | 1.46146E-03 | -8.23121E-04 | 4.52742E-03 | -2.79064E-03 |
| 11.300 | 1.52733E 00 | 1.38164E-03 | -7.73919E-04 | 4.25671E-03 | -2.62520E-03 |
| 11.400 | 1.52746E 00 | 1.30658E-03 | -7.27636E-04 | 4.00205E-03 | -2.46956E-03 |
| 11.500 | 1.52759E 00 | 1.23602E-03 | -6.84099E-04 | 3.76249E-03 | -2.32314E-03 |
| 11.600 | 1.52771E 00 | 1.16968E-03 | -6.43146E-04 | 3.53713E-03 | -2.18539E-03 |
| 11.700 | 1.52782E 00 | 1.10731E-03 | -6.04623E-04 | 3.32514E-03 | -2.05580E-03 |
| 11.800 | 1.52793E 00 | 1.04868E-03 | -5.68386E-04 | 3.12572E-03 | -1.93389E-03 |
| 11.900 | 1.52803E 00 | 9.93558E-04 | -5.34299E-04 | 2.93812E-03 | -1.81920E-03 |
| 12.000 | 1.52813E 00 | 9.41748E-04 | -5.02235E-04 | 2.76165E-03 | -1.71131E-03 |
| 12.100 | 1.52822E 00 | 8.93048E-04 | -4.72074E-04 | 2.59565E-03 | -1.60981E-03 |
| 12.200 | 1.52831E 00 | 8.47274E-04 | -4.43703E-04 | 2.43949E-03 | -1.51433E-03 |
| 12.300 | 1.52839E 00 | 8.04251E-04 | -4.17015E-04 | 2.29259E-03 | -1.42450E-03 |
| 12.400 | 1.52847E 00 | 7.63818E-04 | -3.91911E-04 | 2.15441E-03 | -1.34001E-03 |
| 12.500 | 1.52854E 00 | 7.25819E-04 | -3.68297E-04 | 2.02442E-03 | -1.26052E-03 |
| 12.600 | 1.52861E 00 | 6.90112E-04 | -3.46085E-04 | 1.90215E-03 | -1.18574E-03 |
| 12.700 | 1.52868E 00 | 6.56559E-04 | -3.25190E-04 | 1.78713E-03 | -1.11540E-03 |
| 12.800 | 1.52875E 00 | 6.25032E-04 | -3.05536E-04 | 1.67893E-03 | -1.04923E-03 |
| 12.900 | 1.52881E 00 | 5.95413E-04 | -2.87048E-04 | 1.57715E-03 | -9.86777E-04 |
| 13.000 | 1.52886E 00 | 5.67586E-04 | -2.69658E-04 | 1.48141E-03 | -9.28420E-04 |
| 13.100 | 1.52892E 00 | 5.41447E-04 | -2.53300E-04 | 1.39135E-03 | -8.73335E-04 |
| 13.200 | 1.52897E 00 | 5.16894E-04 | -2.37912E-04 | 1.30664E-03 | -8.21517E-04 |
| 13.300 | 1.52902E 00 | 4.93834E-04 | -2.23438E-04 | 1.22695E-03 | -7.72773E-04 |
| 13.400 | 1.52907E 00 | 4.72177E-04 | -2.09824E-04 | 1.15198E-03 | -7.26919E-04 |
| 13.500 | 1.52912E 00 | 4.51842E-04 | -1.97017E-04 | 1.08147E-03 | -6.83786E-04 |
| 13.600 | 1.52916E 00 | 4.32749E-04 | -1.84970E-04 | 1.01514E-03 | -6.43210E-04 |
| 13.700 | 1.52920E 00 | 4.14824E-04 | -1.73639E-04 | 9.52749E-04 | -6.05042E-04 |

Pr = 0.1000 (Contd.)

| $\eta$ | F           | F'           | F''          | $\phi$      | $\phi'$      |
|--------|-------------|--------------|--------------|-------------|--------------|
| 13.800 | 1.52925E 00 | 3.97999E-04  | -1.62980E-04 | 8.94058E-04 | -5.69138E-04 |
| 13.900 | 1.52928E 00 | 3.82207E-04  | -1.52954E-04 | 8.38850E-04 | -5.35362E-04 |
| 14.000 | 1.52932E 00 | 3.67388E-04  | -1.43522E-04 | 7.86918E-04 | -5.03593E-04 |
| 14.100 | 1.52936E 00 | 3.53484E-04  | -1.34651E-04 | 7.38069E-04 | -4.73707E-04 |
| 14.200 | 1.52939E 00 | 3.40440E-04  | -1.26307E-04 | 6.92118E-04 | -4.45594E-04 |
| 14.300 | 1.52943E 00 | 3.28206E-04  | -1.18457E-04 | 6.48894E-04 | -4.19149E-04 |
| 14.400 | 1.52946E 00 | 3.16733E-04  | -1.11074E-04 | 6.08236E-04 | -3.94273E-04 |
| 14.500 | 1.52949E 00 | 3.05977E-04  | -1.04129E-04 | 5.69990E-04 | -3.70873E-04 |
| 14.600 | 1.52952E 00 | 2.95894E-04  | -9.75962E-05 | 5.34015E-04 | -3.48862E-04 |
| 14.700 | 1.52955E 00 | 2.86445E-04  | -9.14513E-05 | 5.00175E-04 | -3.28156E-04 |
| 14.800 | 1.52958E 00 | 2.77591E-04  | -8.56711E-05 | 4.68343E-04 | -3.08679E-04 |
| 14.900 | 1.52960E 00 | 2.69299E-04  | -8.02341E-05 | 4.38400E-04 | -2.90358E-04 |
| 15.000 | 1.52963E 00 | 2.61534E-04  | -7.51199E-05 | 4.10235E-04 | -2.73124E-04 |
| 15.100 | 1.52966E 00 | 2.54265E-04  | -7.03093E-05 | 3.83741E-04 | -2.56913E-04 |
| 15.200 | 1.52968E 00 | 2.47462E-04  | -6.57843E-05 | 3.58820E-04 | -2.41664E-04 |
| 15.300 | 1.52971E 00 | 2.41099E-04  | -6.15279E-05 | 3.35378E-04 | -2.27319E-04 |
| 15.400 | 1.52973E 00 | 2.35148E-04  | -5.75243E-05 | 3.13328E-04 | -2.13826E-04 |
| 15.500 | 1.52975E 00 | 2.29586E-04  | -5.37583E-05 | 2.92587E-04 | -2.01134E-04 |
| 15.600 | 1.52978E 00 | 2.24389E-04  | -5.02159E-05 | 2.73076E-04 | -1.89195E-04 |
| 15.700 | 1.52980E 00 | 2.19536E-04  | -4.68838E-05 | 2.54724E-04 | -1.77964E-04 |
| 15.800 | 1.52982E 00 | 2.15006E-04  | -4.37495E-05 | 2.37461E-04 | -1.67400E-04 |
| 15.900 | 1.52984E 00 | 2.10780E-04  | -4.08014E-05 | 2.21223E-04 | -1.57463E-04 |
| 16.000 | 1.52986E 00 | 2.06840E-04  | -3.80283E-05 | 2.05949E-04 | -1.48116E-04 |
| 16.100 | 1.52988E 00 | 2.03169E-04  | -3.54198E-05 | 1.91581E-04 | -1.39323E-04 |
| 16.200 | 1.52990E 00 | 1.99751E-04  | -3.29662E-05 | 1.78067E-04 | -1.31052E-04 |
| 16.300 | 1.52992E 00 | 1.96571E-04  | -3.06582E-05 | 1.65355E-04 | -1.23273E-04 |
| 16.400 | 1.52994E 00 | 1.93614E-04  | -2.84873E-05 | 1.53397E-04 | -1.15954E-04 |
| 16.500 | 1.52996E 00 | 1.90869E-04  | -2.64454E-05 | 1.42149E-04 | -1.09071E-04 |
| 16.600 | 1.52998E 00 | 1.88321E-04  | -2.45246E-05 | 1.31569E-04 | -1.02596E-04 |
| 16.700 | 1.53000E 00 | 1.85860E-04  | -2.27179E-05 | 1.21617E-04 | -9.65049E-05 |
| 16.800 | 1.53002E 00 | 1.83774E-04  | -2.10185E-05 | 1.12256E-04 | -9.07757E-05 |
| 16.900 | 1.53004E 00 | 1.81753E-04  | -1.94199E-05 | 1.03451E-04 | -8.53865E-05 |
| 17.000 | 1.53005E 00 | 1.79887E-04  | -1.79163E-05 | 9.51683E-05 | -8.03172E-05 |
| 17.100 | 1.53007E 00 | 1.78167E-04  | -1.65020E-05 | 8.73774E-05 | -7.55488E-05 |
| 17.200 | 1.53009E 00 | 1.76584E-04  | -1.51717E-05 | 8.00491E-05 | -7.10635E-05 |
| 17.300 | 1.53011E 00 | 1.75130E-04  | -1.39203E-05 | 7.31559E-05 | -6.68444E-05 |
| 17.400 | 1.53012E 00 | 1.73797E-04  | -1.27433E-05 | 6.66719E-05 | -6.28758E-05 |
| 17.500 | 1.53014E 00 | 1.72579E-04  | -1.16361E-05 | 6.05728E-05 | -5.91428E-05 |
| 17.600 | 1.53016E 00 | 1.71468E-04  | -1.05947E-05 | 5.48359E-05 | -5.56315E-05 |
| 17.700 | 1.53018E 00 | 1.70458E-04  | -9.61513E-06 | 4.94396E-05 | -5.23284E-05 |
| 17.800 | 1.53019E 00 | 1.69543E-04  | -8.69373E-06 | 4.43637E-05 | -4.92215E-05 |
| 17.900 | 1.53021E 00 | 1.68717E-04  | -7.82705E-06 | 3.95892E-05 | -4.62990E-05 |
| 18.000 | 1.53023E 00 | 1.67976E-04  | -7.01182E-06 | 3.50981E-05 | -4.35500E-05 |
| 18.100 | 1.53024E 00 | 1.67313E-04  | -6.24501E-06 | 3.08737E-05 | -4.09642E-05 |
| 18.200 | 1.53026E 00 | 1.66725E-04  | -5.52374E-06 | 2.69002E-05 | -3.85320E-05 |
| 18.300 | 1.53028E 00 | 1.66207E-04  | -4.84529E-06 | 2.31625E-05 | -3.62441E-05 |
| 18.400 | 1.53029E 00 | 1.65755E-04  | -4.20714E-06 | 1.96468E-05 | -3.40920E-05 |
| 18.500 | 1.53031E 00 | 1.65364E-04  | -3.60688E-06 | 1.63399E-05 | -3.20678E-05 |
| 18.600 | 1.53033E 00 | 1.65032E-04  | -3.04226E-06 | 1.32293E-05 | -3.01636E-05 |
| 18.700 | 1.53034E 00 | 1.64755E-04  | -2.51118E-06 | 1.03034E-05 | -2.83726E-05 |
| 18.800 | 1.53036E 00 | 1.644529E-04 | -2.01163E-06 | 7.55121E-06 | -2.66879E-05 |
| 18.900 | 1.53038E 00 | 1.64352E-04  | -1.54176E-06 | 4.96246E-06 | -2.51032E-05 |
| 19.000 | 1.53039E 00 | 1.64220E-04  | -1.09978E-06 | 2.52744E-06 | -2.36125E-05 |

$\Pr = 1.0000$ 

| $\eta$ | F           | F'          | F''           | $\phi$       | $\phi'$      |
|--------|-------------|-------------|---------------|--------------|--------------|
| 0.     | 0.          | 0.          | 7.21960E-01   | 1.35740E+00  | -1.00000E+00 |
| 0.100  | 3.38782E-03 | 6.55798E-02 | 5.91382E-01   | 1.25756E+00  | -9.95168E-01 |
| 0.200  | 1.26985E-02 | 1.18639E-01 | 4.71677E-01   | 1.15868E+00  | -9.80962E-01 |
| 0.300  | 2.67356E-02 | 1.60298E-01 | 3.63456E-01   | 1.06166E+00  | -9.57909E-01 |
| 0.400  | 4.44171E-02 | 1.91725E-01 | 2.67052E-01   | 9.67369E-01  | -9.26698E-01 |
| 0.500  | 6.47790E-02 | 2.14105E-01 | 1.82524E-01   | 8.76567E-01  | -8.88212E-01 |
| 0.600  | 8.69759E-02 | 2.28619E-01 | 1.09679E-01   | 7.89934E-01  | -8.43516E-01 |
| 0.700  | 1.10279E-01 | 2.36416E-01 | 4.80882E-02   | 7.08031E-01  | -7.93826E-01 |
| 0.800  | 1.34072E-01 | 2.38591E-01 | -2.87288E-03  | 6.31292E-01  | -7.40450E-01 |
| 0.900  | 1.57844E-01 | 2.36170E-01 | -4.39916E-02  | 5.60019E-01  | -6.84727E-01 |
| 1.000  | 1.81184E-01 | 2.30091E-01 | -7.61729E-02  | 4.94381E-01  | -6.27955E-01 |
| 1.100  | 2.03769E-01 | 2.21200E-01 | -1.00396E-01  | 4.34423E-01  | -5.71335E-01 |
| 1.200  | 2.25355E-01 | 2.10243E-01 | -1.17672E-01  | 3.80074E-01  | -5.15924E-01 |
| 1.300  | 2.45770E-01 | 1.97864E-01 | -1.29006E-01  | 3.31168E-01  | -4.62630E-01 |
| 1.400  | 2.64899E-01 | 1.84608E-01 | -1.35364E-01  | 2.87460E-01  | -4.12062E-01 |
| 1.500  | 2.82677E-01 | 1.70927E-01 | -1.37653E-01  | 2.48646E-01  | -3.64802E-01 |
| 1.600  | 2.99082E-01 | 1.57185E-01 | -1.36695E-01  | 2.14380E-01  | -3.21144E-01 |
| 1.700  | 3.14122E-01 | 1.43671E-01 | -1.33227E-01  | 1.84291E-01  | -2.81254E-01 |
| 1.800  | 3.27831E-01 | 1.30602E-01 | -1.27886E-01  | 1.58002E-01  | -2.45161E-01 |
| 1.900  | 3.40263E-01 | 1.18138E-01 | -1.21219E-01  | 1.35135E-01  | -2.12789E-01 |
| 2.000  | 3.51483E-01 | 1.06387E-01 | -1.13679E-01  | 1.15326E-01  | -1.83981E-01 |
| 2.100  | 3.61566E-01 | 9.54187E-02 | -1.05639E-01  | 9.82272E-02  | -1.58524E-01 |
| 2.200  | 3.70594E-01 | 8.52626E-02 | -9.73929E-02  | 8.35175E-02  | -1.36166E-01 |
| 2.300  | 3.78647E-01 | 7.59393E-02 | -8.91721E-02  | 7.08997E-02  | -1.16638E-01 |
| 2.400  | 3.85809E-01 | 6.74254E-02 | -8.111503E-02 | 6.01047E-02  | -9.96656E-02 |
| 2.500  | 3.92159E-01 | 5.96983E-02 | -7.34536E-02  | 5.08905E-02  | -8.49774E-02 |
| 2.600  | 3.97773E-01 | 5.27209E-02 | -6.61690E-02  | 4.30417E-02  | -7.23144E-02 |
| 2.700  | 4.02726E-01 | 4.64489E-02 | -5.93519E-02  | 3.63682E-02  | -6.14399E-02 |
| 2.800  | 4.07085E-01 | 4.08339E-02 | -5.20326E-02  | 3.07030E-02  | -5.21126E-02 |
| 2.900  | 4.10913E-01 | 3.58254E-02 | -4.72219E-02  | 2.59005E-02  | -4.41477E-02 |
| 3.000  | 4.14269E-01 | 3.13727E-02 | -4.19160E-02  | 2.18343E-02  | -3.73571E-02 |
| 3.100  | 4.17205E-01 | 2.74259E-02 | -3.71003E-02  | 1.83953E-02  | -3.15793E-02 |
| 3.200  | 4.19769E-01 | 2.39370E-02 | -3.27527E-02  | 1.54895E-02  | -2.66718E-02 |
| 3.300  | 4.22006E-01 | 2.08607E-02 | -2.88459E-02  | 1.30361E-02  | -2.25096E-02 |
| 3.400  | 4.23954E-01 | 1.81542E-02 | -2.53499E-02  | 1.09663E-02  | -1.89844E-02 |
| 3.500  | 4.25648E-01 | 1.57780E-02 | -2.22330E-02  | 9.22118E-03  | -1.60019E-02 |
| 3.600  | 4.27119E-01 | 1.36960E-02 | -1.94634E-02  | 7.75056E-03  | -1.34813E-02 |
| 3.700  | 4.28396E-01 | 1.18748E-02 | -1.70098E-02  | 6.51186E-03  | -1.13528E-02 |
| 3.800  | 4.29502E-01 | 1.02845E-02 | -1.48419E-02  | 5.46893E-03  | -9.55669E-03 |
| 3.900  | 4.30459E-01 | 8.89789E-03 | -1.29312E-02  | 4.59115E-03  | -8.04216E-03 |
| 4.000  | 4.31287E-01 | 7.69060E-03 | -1.12509E-02  | 3.85257E-03  | -6.76574E-03 |
| 4.100  | 4.32003E-01 | 6.64085E-03 | -9.77636E-03  | 3.23130E-03  | -5.69053E-03 |
| 4.200  | 4.32620E-01 | 5.72923E-03 | -8.48474E-03  | 2.70881E-03  | -4.78519E-03 |
| 4.300  | 4.33152E-01 | 4.93850E-03 | -7.35531E-03  | 2.26949E-03  | -4.02316E-03 |
| 4.400  | 4.33611E-01 | 4.25339E-03 | -6.36928E-03  | 1.90015E-03  | -3.38196E-03 |
| 4.500  | 4.34006E-01 | 3.66043E-03 | -5.50971E-03  | 1.58970E-03  | -2.84257E-03 |
| 4.600  | 4.34346E-01 | 3.14774E-03 | -4.76140E-03  | 1.32878E-03  | -2.38893E-03 |
| 4.700  | 4.34638E-01 | 2.70489E-03 | -4.11078E-03  | 1.10951E-03  | -2.00749E-03 |
| 4.800  | 4.34889E-01 | 2.32273E-03 | -3.54576E-03  | 9.25261E-04  | -1.68681E-03 |
| 4.900  | 4.35104E-01 | 1.99325E-03 | -3.05562E-03  | 7.70447E-04  | -1.41726E-03 |
| 5.000  | 4.35289E-01 | 1.70943E-03 | -2.63083E-03  | 6.40377E-04  | -1.19070E-03 |
| 5.100  | 4.35544E-01 | 1.46517E-03 | -2.26317E-03  | 5.31102E-04  | -1.00031E-03 |
| 5.200  | 4.35583E-01 | 1.25514E-03 | -1.94512E-03  | 4.39302E-04  | -8.40325E-04 |
| 5.300  | 4.35700E-01 | 1.07471E-03 | -1.67026E-03  | 3.62186E-04  | -7.05898E-04 |
| 5.400  | 4.35799E-01 | 9.19838E-04 | -1.43291E-03  | 2.97407E-04  | -5.92956E-04 |
| 5.500  | 4.35884E-01 | 7.87039E-04 | -1.22812E-03  | 2.42993E-04  | -4.98070E-04 |
| 5.600  | 4.35957E-01 | 6.73274E-04 | -1.05154E-03  | 1.97288E-04  | -4.18358E-04 |
| 5.700  | 4.36020E-01 | 5.75917E-04 | -8.99388E-04  | 1.58897E-04  | -3.51396E-04 |
| 5.800  | 4.36073E-01 | 4.92693E-04 | -7.68374E-04  | 1.26652E-04  | -2.95146E-04 |
| 5.900  | 4.36118E-01 | 4.21634E-04 | -5.56360E-04  | 9.95684E-05  | -2.47897E-04 |
| 6.000  | 4.36158E-01 | 3.61041E-04 | -5.58636E-04  | 7.68208E-05  | -2.08209E-04 |
| 6.100  | 4.36191E-01 | 3.09448E-04 | -4.75313E-04  | 5.77151E-05  | -1.74873E-04 |
| 6.200  | 4.36220E-01 | 2.65587E-04 | -4.03706E-04  | 4.16684E-05  | -1.46873E-04 |
| 6.300  | 4.36244E-01 | 2.28369E-04 | -3.42219E-04  | 2.81912E-05  | -1.23356E-04 |
| 6.400  | 4.36266E-01 | 1.96853E-04 | -2.89446E-04  | 1.68720E-05  | -1.03603E-04 |
| 6.500  | 4.36284E-01 | 1.70230E-04 | -2.44174E-04  | 7.36530E-06  | -8.70125E-05 |
| 6.600  | 4.36300E-01 | 1.47804E-04 | -2.05354E-04  | -6.19002E-07 | -7.30785E-05 |
| 6.700  | 4.36314E-01 | 1.28975E-04 | -1.72080E-04  | -7.32470E-06 | -6.13756E-05 |
| 6.800  | 4.36326E-01 | 1.13229E-04 | -1.43573E-04  | -1.29565E-05 | -5.15467E-05 |

**Pr = 1.0000 (Contd.)**

| $\eta$ | F           | F'          | F''          | $\phi$       | $\phi'$      |
|--------|-------------|-------------|--------------|--------------|--------------|
| 6.900  | 4.36336E-01 | 1.00124E-04 | -1.19158E-04 | -1.76864E-05 | -4.32916E-05 |
| 7.000  | 4.36346E-01 | 8.92801E-05 | -9.82562E-05 | -2.16589E-05 | -3.63585E-05 |
| 7.100  | 4.36354E-01 | 8.03721E-05 | -8.03685E-05 | -2.49951E-05 | -3.05356E-05 |
| 7.200  | 4.36362E-01 | 7.31204E-05 | -6.50655E-05 | -2.77970E-05 | -2.56452E-05 |
| 7.300  | 4.36369E-01 | 6.72852E-05 | -5.19783E-05 | -3.01502E-05 | -2.15380E-05 |
| 7.400  | 4.36375E-01 | 6.26615E-05 | -4.07896E-05 | -3.21265E-05 | -1.80886E-05 |
| 7.500  | 4.36381E-01 | 5.90732E-05 | -3.12270E-05 | -3.37863E-05 | -1.51916E-05 |
| 7.600  | 4.36387E-01 | 5.63697E-05 | -2.30566E-05 | -3.51803E-05 | -1.27586E-05 |
| 7.700  | 4.36393E-01 | 5.44221E-05 | -1.60779E-05 | -3.63510E-05 | -1.07152E-05 |
| 7.800  | 4.36398E-01 | 5.31202E-05 | -1.01186E-05 | -3.73342E-05 | -8.99910E-06 |
| 7.900  | 4.36403E-01 | 5.23694E-05 | -5.03130E-06 | -3.81600E-05 | -7.55787E-06 |
| 8.000  | 4.36409E-01 | 5.20891E-05 | -6.89492E-07 | -3.88535E-05 | -6.34746E-06 |

**Pr = 10.0000**

| $\eta$ | F           | F'          | F''          | $\phi$       | $\phi'$      |
|--------|-------------|-------------|--------------|--------------|--------------|
| 0.     | 0.          | 0.          | 3.06390E-01  | 7.67460E-01  | -1.00000E 00 |
| 0.100  | 1.40822E-03 | 2.69689E-02 | 2.34664E-01  | 6.67853E-01  | -9.88215E-01 |
| 0.200  | 5.17144E-03 | 4.72671E-02 | 1.72960E-01  | 5.70600E-01  | -9.52992E-01 |
| 0.300  | 1.06724E-02 | 6.18875E-02 | 1.21044E-01  | 4.77999E-01  | -8.95503E-01 |
| 0.400  | 1.73916E-02 | 7.17858E-02 | 7.84154E-02  | 3.92146E-01  | -8.18625E-01 |
| 0.500  | 2.49021E-02 | 7.78557E-02 | 4.43359E-02  | 3.14759E-01  | -7.27128E-01 |
| 0.600  | 3.28622E-02 | 8.09075E-02 | 1.78870E-02  | 2.47011E-01  | -6.26920E-01 |
| 0.700  | 4.10068E-02 | 8.16531E-02 | -1.96529E-03 | 1.89449E-01  | -5.24471E-01 |
| 0.800  | 4.91363E-02 | 8.06984E-02 | -1.62963E-02 | 1.41991E-01  | -4.25754E-01 |
| 0.900  | 5.71066E-02 | 7.85428E-02 | -2.61546E-02 | 1.04014E-01  | -3.35520E-01 |
| 1.000  | 6.48182E-02 | 7.55842E-02 | -3.25070E-02 | 7.44986E-02  | -2.56881E-01 |
| 1.100  | 7.22070E-02 | 7.21299E-02 | -3.62012E-02 | 5.22017E-02  | -1.91265E-01 |
| 1.200  | 7.92354E-02 | 6.84088E-02 | -3.79478E-02 | 3.58109E-02  | -1.38655E-01 |
| 1.300  | 8.58854E-02 | 6.45860E-02 | -3.83180E-02 | 2.40718E-02  | -9.79908E-02 |
| 1.400  | 9.21530E-02 | 6.07762E-02 | -3.77540E-02 | 1.58700E-02  | -6.76008E-02 |
| 1.500  | 9.80436E-02 | 5.70553E-02 | -3.65858E-02 | 1.02726E-02  | -4.55839E-02 |
| 1.600  | 1.03569E-01 | 5.34711E-02 | -3.50519E-02 | 6.53624E-03  | -3.00838E-02 |
| 1.700  | 1.08743E-01 | 5.00515E-02 | -3.33188E-02 | 4.09371E-03  | -1.94566E-02 |
| 1.800  | 1.13585E-01 | 4.68102E-02 | -3.14992E-02 | 2.52802E-03  | -1.23465E-02 |
| 1.900  | 1.18112E-01 | 4.37520E-02 | -2.96667E-02 | 1.54272E-03  | -7.69610E-03 |
| 2.000  | 1.22341E-01 | 4.08757E-02 | -2.78678E-02 | 9.33321E-04  | -4.71757E-03 |
| 2.100  | 1.26293E-01 | 3.81764E-02 | -2.61302E-02 | 5.62485E-04  | -2.84663E-03 |
| 2.200  | 1.29982E-01 | 3.56471E-02 | -2.44649E-02 | 3.40246E-04  | -1.69242E-03 |
| 2.300  | 1.33427E-01 | 3.32797E-02 | -2.28929E-02 | 2.08967E-04  | -9.92228E-04 |
| 2.400  | 1.36643E-01 | 3.10656E-02 | -2.14033E-02 | 1.324469E-04 | -5.74030E-04 |
| 2.500  | 1.39645E-01 | 2.89962E-02 | -2.00002E-02 | 8.84718E-05  | -3.27862E-04 |
| 2.600  | 1.42447E-01 | 2.70628E-02 | -1.86812E-02 | 6.34856E-05  | -1.84910E-04 |
| 2.700  | 1.45052E-01 | 2.52572E-02 | -1.74434E-02 | 4.94750E-05  | -1.02948E-04 |
| 2.800  | 1.47503E-01 | 2.35715E-02 | -1.62828E-02 | 4.17228E-05  | -5.65168E-05 |
| 2.900  | 1.49780E-01 | 2.19982E-02 | -1.51956E-02 | 3.74975E-05  | -3.05141E-05 |
| 3.000  | 1.51906E-01 | 2.05301E-02 | -1.41778E-02 | 3.52373E-05  | -1.61148E-05 |
| 3.100  | 1.53890E-01 | 1.91605E-02 | -1.32255E-02 | 3.40597E-05  | -8.23143E-06 |
| 3.200  | 1.55741E-01 | 1.78830E-02 | -1.23349E-02 | 3.34717E-05  | -3.96789E-06 |
| 3.300  | 1.57469E-01 | 1.66916E-02 | -1.15023E-02 | 3.32006E-05  | -1.69488E-06 |
| 3.400  | 1.59082E-01 | 1.55807E-02 | -1.07242E-02 | 3.30970E-05  | -5.05536E-07 |
| 3.500  | 1.60588E-01 | 1.45450E-02 | -9.99736E-03 | 3.30802E-05  | 9.97625E-08  |
| 3.600  | 1.61993E-01 | 1.35796E-02 | -9.31853E-03 | 3.31067E-05  | 3.93694E-07  |
| 3.700  | 1.63306E-01 | 1.26798E-02 | -8.68474E-03 | 3.31536E-05  | 5.23726E-07  |
| 3.800  | 1.64531E-01 | 1.18412E-02 | -8.09316E-03 | 3.32087E-05  | 5.68908E-07  |
| 3.900  | 1.65676E-01 | 1.10599E-02 | -7.54112E-03 | 3.32660E-05  | 5.71230E-07  |
| 4.000  | 1.66745E-01 | 1.03318E-02 | -7.02610E-03 | 3.33223E-05  | 5.52784E-07  |

Pr = 100.0000

| $\eta$ | F           | F'          | F''          | $\phi$      | $\phi'$      |
|--------|-------------|-------------|--------------|-------------|--------------|
| 0.     | 0.          | 0.          | 1.26200E-01  | 4.65660E-01 | -1.00000E 00 |
| 0.100  | 5.57549E-04 | 1.04580E-02 | 8.46141E-02  | 3.66652E-01 | -9.70140E-01 |
| 0.200  | 1.96933E-03 | 1.72460E-02 | 5.27046E-02  | 2.73617E-01 | -8.81081E-01 |
| 0.300  | 3.91552E-03 | 2.12931E-02 | 2.96028E-02  | 1.92107E-01 | -7.42329E-01 |
| 0.400  | 6.16395E-03 | 2.34150E-02 | 1.39425E-02  | 1.26091E-01 | -5.75623E-01 |
| 0.500  | 8.55663E-03 | 2.42747E-02 | 4.07536E-03  | 7.69879E-02 | -4.08608E-01 |
| 0.600  | 1.09935E-02 | 2.43676E-02 | -1.65816E-03 | 4.35827E-02 | -2.64778E-01 |
| 0.700  | 1.34160E-02 | 2.40325E-02 | -4.69607E-03 | 2.28334E-02 | -1.56501E-01 |
| 0.800  | 1.57929E-02 | 2.34814E-02 | -6.12934E-03 | 1.10673E-02 | -8.44229E-02 |
| 0.900  | 1.81092E-02 | 2.28354E-02 | -6.69086E-03 | 4.97050E-03 | -4.16214E-02 |
| 1.000  | 2.03590E-02 | 2.21574E-02 | -6.82039E-03 | 2.07909E-03 | -1.87876E-02 |
| 1.100  | 2.25407E-02 | 2.14777E-02 | -6.75520E-03 | 8.21805E-04 | -7.77928E-03 |
| 1.200  | 2.46549E-02 | 2.08090E-02 | -6.61106E-03 | 3.19649E-04 | -2.95929E-03 |
| 1.300  | 2.67030E-02 | 2.01563E-02 | -6.43933E-03 | 1.35248E-04 | -1.03429E-03 |
| 1.400  | 2.86867E-02 | 1.95213E-02 | -6.26076E-03 | 7.30800E-05 | -3.30487E-04 |
| 1.500  | 3.06078E-02 | 1.89041E-02 | -6.08300E-03 | 5.40380E-05 | -9.42613E-05 |
| 1.600  | 3.24681E-02 | 1.83066E-02 | -5.90863E-03 | 4.89802E-05 | -2.14126E-05 |
| 1.700  | 3.42693E-02 | 1.77223E-02 | -5.73840E-03 | 4.80859E-05 | -8.80647E-07 |
| 1.800  | 3.60132E-02 | 1.71568E-02 | -5.57249E-03 | 4.83159E-05 | 4.25384E-06  |
| 1.900  | 3.77012E-02 | 1.66076E-02 | -5.41089E-03 | 4.88056E-05 | 5.22532E-06  |
| 2.000  | 3.93352E-02 | 1.60745E-02 | -5.25354E-03 | 4.93291E-05 | 5.17535E-06  |
| 2.100  | 4.09166E-02 | 1.55568E-02 | -5.10038E-03 | 4.98341E-05 | 4.91222E-06  |
| 2.200  | 4.24471E-02 | 1.50542E-02 | -4.95132E-03 | 5.03109E-05 | 4.62595E-06  |
| 2.300  | 4.39280E-02 | 1.45664E-02 | -4.80630E-03 | 5.07598E-05 | 4.35568E-06  |
| 2.400  | 4.53608E-02 | 1.40928E-02 | -4.66524E-03 | 5.11828E-05 | 4.10682E-06  |
| 2.500  | 4.67470E-02 | 1.36332E-02 | -4.52805E-03 | 5.15819E-05 | 3.87812E-06  |
| 2.600  | 4.80879E-02 | 1.31871E-02 | -4.39467E-03 | 5.19590E-05 | 3.66740E-06  |
| 2.700  | 4.93849E-02 | 1.27542E-02 | -4.26501E-03 | 5.23159E-05 | 3.47264E-06  |
| 2.800  | 5.06392E-02 | 1.23340E-02 | -4.13900E-03 | 5.26540E-05 | 3.29208E-06  |
| 2.900  | 5.18521E-02 | 1.19262E-02 | -4.01656E-03 | 5.29747E-05 | 3.12421E-06  |
| 3.000  | 5.30248E-02 | 1.15306E-02 | -3.89762E-03 | 5.32792E-05 | 2.96774E-06  |
| 3.100  | 5.41586E-02 | 1.11466E-02 | -3.78209E-03 | 5.35686E-05 | 2.82155E-06  |
| 3.200  | 5.52545E-02 | 1.07740E-02 | -3.66989E-03 | 5.38439E-05 | 2.68469E-06  |
| 3.300  | 5.63138E-02 | 1.04125E-02 | -3.56096E-03 | 5.41058E-05 | 2.55630E-06  |
| 3.400  | 5.73374E-02 | 1.00617E-02 | -3.45521E-03 | 5.43554E-05 | 2.43563E-06  |
| 3.500  | 5.83265E-02 | 9.72137E-03 | -3.35257E-03 | 5.45932E-05 | 2.32204E-06  |
| 3.600  | 5.92820E-02 | 9.39112E-03 | -3.25297E-03 | 5.48200E-05 | 2.21494E-06  |
| 3.700  | 6.02050E-02 | 9.07068E-03 | -3.15632E-03 | 5.50364E-05 | 2.11381E-06  |
| 3.800  | 6.10965E-02 | 8.75976E-03 | -3.06256E-03 | 5.52429E-05 | 2.01819E-06  |
| 3.900  | 6.19573E-02 | 8.45807E-03 | -2.97162E-03 | 5.54402E-05 | 1.92765E-06  |
| 4.000  | 6.27884E-02 | 8.16534E-03 | -2.88341E-03 | 5.56286E-05 | 1.84184E-06  |

## APPENDIX II

## INTEGRATION OF EQ. (43)

Since a series solution is required in the analysis, the integration can be carried out by dividing the integral in Eq. (43) into two parts:

$$\begin{aligned}
 U^{(1)}(X, 0) &= -\frac{4 F(\infty)}{\pi 5^{1/5}} \int_0^1 \frac{d\zeta}{\zeta^{1/5}(X - \zeta)} \\
 &= -\frac{4 F(\infty)}{\pi 5^{1/5}} \left[ \int_0^X \frac{d\zeta}{\zeta^{1/5} X \left(1 - \frac{\zeta}{X}\right)} - \int_X^1 \frac{d\zeta}{\zeta^{6/5} \left(1 - \frac{X}{\zeta}\right)} \right] \\
 &= -\frac{4 F(\infty)}{\pi 5^{1/5}} \left[ \int_0^X \frac{1}{X \zeta^{1/5}} \left\{ 1 + \frac{\zeta}{X} + \left(\frac{\zeta}{X}\right)^2 + \dots \right\} d\zeta \right. \\
 &\quad \left. - \int_X^1 \frac{1}{\zeta^{6/5}} \left\{ 1 + \frac{X}{\zeta} + \left(\frac{X}{\zeta}\right)^2 + \dots \right\} d\zeta \right] \\
 &= \frac{20 F(\infty)}{\pi(5X)^{1/5}} \left[ \sum_{m=0}^{\infty} \frac{1}{5 m + 1} - \sum_{m=0}^{\infty} \frac{1}{5 m + 4} - \sum_{m=0}^{\infty} \frac{X^{m+1/5}}{5 m + 1} \right]
 \end{aligned}$$

which is Eq. (44).

## VII. NOMENCLATURE

|                                |  |
|--------------------------------|--|
| $C_p$                          | heat capacity at constant pressure per unit mass,<br>$L^2/t^2T$ .                    |
| $C_v$                          | heat capacity at constant volume per unit mass,<br>$L^2/t^2T$ .                      |
| $F(\eta)$                      | zeroth-order velocity function, dimensionless.                                       |
| $F(\infty)$                    | asymptotic value of $F(\eta)$ at edge of boundary layer,<br>dimensionless.           |
| $F', F'', F'''$                | derivatives of $F(\eta)$ with respect to $\eta$ , dimensionless.                     |
| $f_{00}(\eta), f_m(\eta)$      | first-order velocity functions, dimensionless.                                       |
| $f_{00}', f_{00}'', f_{00}'''$ | derivatives of $f_{00}(\eta)$ with respect to $\eta$ , dimensionless.                |
| $f_m', f_m'', f_m'''$          | derivatives of $f_m(\eta)$ with respect to $\eta$ , dimensionless.                   |
| $G$                            | dimensionless gravitational acceleration defined by<br>Eq. (15)                      |
| $Gr_L^*$                       | modified Grashof number based on $L$ , $g \beta L^4 q / k \nu^2$ ,<br>dimensionless. |
| $Gr_x^*$                       | modified Grashof number based on $x$ , $g \beta x^4 q / k \nu^2$ ,<br>dimensionless. |
| $g$                            | gravitational acceleration, $L/t^2$ .  |
| $g_i$                          | body force vector per unit mass, $L/t^2$ .   |
| $i$                            | $\sqrt{-1}$ .  |
| $i, j$                         | indices.   |
| $k$                            | thermal conductivity, $ML/t^3T$ .  |
| $L$                            | vertical length of plate, $L$ .  |
| $m$                            | subscript for 0, 1, 2, . . .   |
| $O( )$                         | order of magnitude.  |
| $P$                            | dimensionless pressure defined by Eq. (15).  |

|                    |   |
|--------------------|---|
| $P^{(0)}, P^{(1)}$ | zeroth- and first-order pressure of $P$ , dimensionless.                    |
| $P_X, P_Y$         | $\partial P / \partial X, \partial P / \partial Y$ .                        |
| $Pr$               | Prandtl number, $\nu/\alpha$ , dimensionless.                               |
| $p$                | pressure, $M/Lt^2$ .  |
| $p_\infty$         | static pressure of undisturbed environment, $M/Lt^2$ .                      |
| $q_j$              | heat flux vector, $M/t^3$ .   |
| $q$                | surface heat flux, $M/t^3$ .  |
| $S$                | 0.864806 . . .  |
| $T$                | absolute temperature, $T$ .   |
| $T_w$              | surface temperature of plate, $T$ .   |
| $T_\infty$         | bulk temperature of undisturbed environment, $T$ .                          |
| $T^*$              | reference temperature, $T$ .  |
| $t$                | time, $t$ .   |
| $U$                | vertical component of dimensionless velocity,<br>$L u / \nu Gr_*^{2/5}$ .   |
| $U^{(0)}, U^{(1)}$ | zeroth- and first-order velocity of $U$ , dimensionless.                    |
| $U_e$              | vertical velocity at edge of boundary layer,<br>dimensionless.              |
| $U_e^{(1)}$        | first-order velocity of $U_e$ , dimensionless.                              |
| $U_p$              | velocity $U$ in potential-flow field, dimensionless.                        |
| $u$                | vertical velocity component, $\psi_y, L/t$ .                                |
| $u$                | dimensionless vertical velocity component, $\psi_y$ .                       |
| $v$                | horizontal component of dimensionless velocity,<br>$L v / \nu Gr_*^{2/5}$ . |
| $v^{(0)}, v^{(1)}$ | zeroth- and first-order velocity of $V$ , dimensionless.                    |

|             |  |
|-------------|--|
| $V_e$       | horizontal velocity at edge of boundary layer,<br>dimensionless.                                       |
| $V_e^{(1)}$ | first-order velocity of $V_e$ , dimensionless.   |
| $V_p$       | velocity $V$ in potential-flow field, dimensionless.   |
| $\nabla_i$  | velocity vector, $L/t$ .   |
| $v$         | horizontal velocity component, $-\psi_x$ , $L/t$ .   |
| $\bar{v}$   | dimensionless horizontal velocity component, $-\bar{\psi}_{\bar{x}}$ .                                 |
| $\hat{v}$   | volume per unit mass, $L^3/M$ .  |
| $X$         | transformed dimensionless vertical coordinate de-<br>fined by Eq. (17).                                |
| $x$         | vertical coordinate, $L$ .   |
| $\bar{x}$   | dimensionless vertical coordinate, $x/L$ .   |
| $Y$         | transformed dimensionless horizontal coordinate de-<br>fined by Eq. (17).                              |
| $y$         | horizontal coordinate, $L$ .   |
| $\bar{y}$   | dimensionless horizontal coordinate, $y/L$ .   |
| $z$         | complex number, $X + i\bar{y}$ .   |
| $\alpha$    | thermal diffusivity, $k/C_p \rho$ , $L^2/t$ .  |
| $\beta$     | thermal coefficient of volumetric expansion, $-(1/\rho)$<br>$(\partial \rho / \partial T)_p$ , $1/T$ . |
| $\beta^*$   | $\beta$ at reference temperature, $1/T$ .  |
| $\epsilon$  | constant parameter in series expansion, $Gr_L^{*-1/5}$ ,<br>dimensionless.                             |
| $\zeta$     | dummy variable for integration, dimensionless.   |
| $\eta$      | similarity variable, $Y/(5X)^{1/5}$ or $(y/x)(Gr_x^*/5)^{1/5}$ ,<br>dimensionless.                     |
| $\Theta$    | dimensionless temperature defined by Eq. (15).   |

|  |  |
|--|--|
| $\theta^{(0)}, \theta^{(1)}$             | zeroth- and first-order temperature of $\theta$ , dimensionless.                               |
| $\theta_X, \theta_Y, \theta_{XX}, \dots$ | $\partial\theta/\partial X, \partial\theta/\partial Y, \partial^2\theta/\partial X^2, \dots$   |
| $\theta_{00}(\eta), \theta_m(\eta)$      | first-order temperature functions, dimensionless.  |
| $\theta'_{00}, \theta''_{00}$            | derivatives of $\theta_{00}(\eta)$ with respect to $\eta$ , dimensionless.                     |
| $\theta'_m, \theta''_m$                  | derivatives of $\theta_m(\eta)$ with respect to $\eta$ , dimensionless.                        |
| $\mu$                                    | viscosity, $M/Lt$ .  |
| $\nu$                                    | kinematic viscosity, $\mu/\rho, L^2/t$ .   |
| $\pi$                                    | 3.14159 . . .  |
| $\rho$                                   | density, $M/L^3$ .   |
| $\rho^*$                                 | $\rho$ at reference temperature, $M/L^3$ .   |
| $\mathfrak{T}_1^j$                       | stress tensor, $M/t^2 L$ .   |
| $\Phi_V$                                 | viscous dissipation function, $1/t^2$ .  |
| $\phi(\eta)$                             | zeroth-order temperature function, dimensionless.  |
| $\phi', \phi''$                          | derivatives of $\phi(\eta)$ with respect to $\eta$ , dimensionless.                            |
| $\Psi$                                   | dimensionless function defined by Eq. (17).  |
| $\Psi^{(0)}, \Psi^{(1)}$                 | zeroth- and first-order $\Psi$ , dimensionless.  |
| $\Psi_X, \Psi_Y, \Psi_{XY}, \dots$       | $\partial\Psi/\partial X, \partial\Psi/\partial Y, \partial^2\Psi/\partial X\partial Y, \dots$ |
| $\psi$                                   | stream function, $L^2/t$ .   |
| $\psi_x, \psi_u$                         | $\partial\psi/\partial x, \partial\psi/\partial y$ .   |
| $\bar{\psi}$                             | dimensionless stream function defined by Eq. (15).   |
| ,  | covariant differentiation.   |

### VIII. BIBLIOGRAPHY

1. Acrivos, A., On the Solution of the Convection Equation in Laminar Boundary Layer Flows, Chem. Eng. Science, 17, 457-465 (1962).
2. Bayley, F. J., Milne, P. A., and Stoddart, D. E., Heat Transfer by Free Convection in a Liquid Metal, Proc. Roy. Soc. (London), 265, 97-108 (1961).
3. Bird, R. B., Stewart, W. E., and Lightfoot, E. N., Transport Phenomena, John Wiley, New York (1962).
4. Bobco, R. P., A Closed-form Solution for Laminar Free Convection on a Vertical Plate with Prescribed, Nonuniform, Wall Heat Flux, J. Aero. Space Sciences, 26, No. 12, 846-847 (1959).
5. Chung, P. M., and Anderson, A. D., Unsteady Laminar Free Convection, J. Heat Transfer, Trans. ASME, Ser. C., 83, 473-478 (1961).
6. Dotson, J. P., Heat Transfer from a Vertical Plate by Free Convection, M. S. Thesis, Purdue University, W. Lafayette, Ind. (1954).
7. Eckert, E. R. G., and Jackson, T. W., Analysis of Turbulent Free-convection Boundary Layer on a Flat Plate, NACA Report 1015 (1951).
8. Eckert, E. R. G., and Soehnghen, E., Studies on Heat-Transfer in Laminar Free Convection with the Zehnder-Mach Interferometer, USAF, Air Materiel Command, Dayton, Ohio, Tech. Report No. 5747 (1948).
9. Eckert, E. R. G., and Soehnghen, E., Interferometric Studies on the Stability and Transition to Turbulence of a Free-convection Boundary Layer, Proc. of Gen. Discuss. on Heat Trans., 321-323, Inst. Mech. Eng., London, and ASME, New York (1951).
10. Eichhorn, R., The Effect of Mass Transfer on Free Convection, J. Heat Transfer, Trans. ASME, Ser. C, 82, 260-263 (1960).
11. Elshin, K. V., Priblizhennoe Reshenie Uravnenii Svabodnoi Konvektsii u Vertikal'noi Neizotermicheskoi Stenki (An Approximate Solution of Equations for Free Convection of a Liquid near a Vertical Non-isothermal Wall), Inzhenerno-Fizicheskii Zhurnal, 4, No. 4, 62-68 (1961). (in Russian)
12. Finston, M., Free Convection Past a Vertical Plate, ZAMP, 7, 527-529 (1956).

13. Foote, J. R., An Asymptotic Method for Free Convection Past a Vertical Plate, ZAMP, 9, 64-67 (1958).
14. Fujii, T., Mathematical Analysis of Heat-Transfer from a Vertical Flat Surface by Laminar Free Convection, Bullet. of Japan Soc. of Mech. Eng., 2, No. 7, 365-369 (1959).
15. Fujii, T., An Analysis of Turbulent Free Convection Heat Transfer from a Vertical Surface, Bullet. of Japan Soc. of Mech. Eng., 2, No. 8, 559-563 (1959).
16. Gebhart, B., Heat Transfer, 251-273, McGraw-Hill Book Co., Inc., New York (1961).
17. Gebhart, B., On Inflexion Points in Natural Convection Profiles, J. Aero. Space Sciences, 29, 485-486 (1962).
18. Gebhart, B., Effects of Viscous Dissipation in Natural Convection, J. Fluid Mechanics, 14, 225-232 (1962).
19. Goldstein, R. J., and Eckert, E. R. G., The Steady and Transient Free Convection Boundary Layer on a Uniformly Heated Vertical Plate, Int. J. Heat Mass Transfer 1, 208-218 (1960).
20. Goldstein, S., Modern Developments in Fluid Dynamics, II, 641, Oxford University Press (1957).
21. Griffiths, E., and Davis, A. H., The Transmission of Heat by Radiation and Convection, Dept. of Scientific and Industrial Research, Food Investigation Board, Special Report, No. 9, His Majesty's Stationery Office, London (1922).
22. Kimball, W. S., and King, W. J., Theory of Heat Conduction and Convection from a Low Hot Vertical Plate, Phil. Mag., 13, Seventh Ser., 888-906 (1932).
23. King, W. J., The Basic Laws and Data of Heat Transmission, Part III - Free Convection, Mechanical Eng., 54, 347-353 (1932).
24. Klyachko, L. S., Relations for the Critical State Describing Transition from Laminar to Turbulent Flow in Free Convection, Int. J. Heat Mass Transfer, 5, 763-764 (1962).
25. Kuo, Y. H., On the Flow of an Incompressible Viscous Fluid Past a Flat Plate at Moderate Reynolds Numbers, J. Math. Phys., 32, 83-101 (1953).

26. Kutateladze, S. S., Borishanskii, V. M., and Novikov, I. I., Heat Transfer in Liquid Metals, J. Nucl. Energy II, 9, 214-229 (1959).
27. Lighthill, M. J., A Technique for Rendering Approximate Solutions to Physical Problem Uniformly Valid, Phil. Mag., 40, 1179-1201 (1949).
28. Lorenz, H., Die Wärmeübertragung an einer ebenen senkrechten Platte an Öl bei natürlicher Konvektion, Z. techn. Physik, No. 9, 362-366 (1934).
29. Lorenz, L., Wiedemanns Annalen, 13, 582 (1881).
30. McAdams, W. H., Heat Transmission, 3rd ed., 165-183, McGraw-Hill Book Co., Inc., New York (1954).
31. Menold, E. R., and Yang, K. T., Asymptotic Solutions for Unsteady Laminar Free Convection on a Vertical Plate, J. Appl. Mech., Trans. ASME, Ser. E., 29, 124-126 (1962).
32. Numan, F., and Pohlhausen, K., Remarks on the Paper by M. Finston: Free Convection Past a Vertical Plate, ZAMP, 9, 67-69 (1958).
33. Nusselt, W., and Jürges, W., VDI-Zeitschr., 72, 597 (1928).
34. Ostrach, S., An Analysis of Laminar Free-Convection Flow and Heat Transfer About a Flat Plate Parallel to the Direction of the Generating Body Force, NACA Tech. Report 1111 (1953).
35. Pai, S. I., Viscous Flow Theory, 1, 115-117, D. Van Nostrand Co., Inc. (1956).
36. Pohlhausen, E., Der Wärmeaustausch zwischen festen Körpern und Flüssigkeiten mit kleiner Reibung und kleiner Wärmeleitung, ZAMM, 1, 115-121 (1921).
37. Saunders, O. A., The Effect of Pressure Upon Natural Convection in Air, Proc. Roy Soc. (London), A157, 278-291 (1936).
38. Saunders, O. A., Natural Convection in Liquids, Proc. Roy. Soc. (London), A172, 55-71 (1939).
39. Scherberg, M. G., Natural Convection Near and Above Thermal Leading Edges on Vertical Walls, Int. J. Heat Mass Transfer, 5, 1001-1010 (1962).

40. Schlichting, H., Grenzschicht-Theorie, 303-310, Karlsruhe (1958).
41. Schmidt, E., Schlierenaufnahmen der Temperaturfelder in der Nähe wärmeabgebender Körper, Forsch. Ing.-Wes., 3, 181 (1932).
42. Schmidt, E., and Beckmann, W., Das Temperatur- und Geschwindigkeitsfeld vor einer Wärme abgebenden senkrechten Platte bei natürlicher Konvektion, Tech. Mech. u. Thermodynamik, Bd. 1, No. 10, 341-349 (1930); cont. Bd. 1, No. 11, 391-406 (1930).
43. Schuh, H., Einige Probleme bei freier Strömung zäher Flüssigkeiten, Göttinger Monographien, Bd. B, Grenzschichten (1946).
44. Siegel, R., Transient Free Convection from a Vertical Flat Plate, Trans. ASME, 80, 347-359 (1958).
45. Slattery, J. C., The Flow External to a Non-Newtonian Boundary Layer, Chem. Eng. Science, 17, 689-691 (1962).
46. Sparrow, E. M., and Cess, R. D., Free Convection with Blowing or Suction, J. Heat Transfer, Trans. ASME, Ser. C, 83, 387-389 (1961).
47. Sparrow, E. M., and Gregg, J. L., Laminar Free Convection from a Vertical Plate with Uniform Surface Heat Flux, Trans. ASME, 78, 435-440 (1956).
48. Sparrow, E. M., and Gregg, J. L., Similar Solutions for Free Convection from a Nonisothermal Vertical Plate, Trans. ASME, 80, 379-386 (1958).
49. Sparrow, E. M., and Gregg, J. L., The Variable Fluid-Property Problem in Free Convection, Trans. ASME, 80, 879-886 (1958).
50. Sparrow, E. M., and Gregg, J. L., Nearly Quasi-Steady Free Convection Heat Transfer in Gases, J. Heat Transfer, Trans. ASME, Ser. C, 82, 258-260 (1960).
51. Sugawara, S., and Michiyoshi, I., The Heat Transfer by Natural Convection in the Unsteady State on a Vertical Flat Wall, Proc. of the First Japan National Congress for Applied Mechanics (1951), National Committee for Theoretical and Applied Mechanics, Science Council of Japan, 501-506 (May 1952).
52. Szewczyk, A. A., Stability and Transition of the Free-convection Layer along a Vertical Flat Plate, Int. J. Heat Mass Transfer, 5, 903-914 (1962).

53. Tsien, H. S., The Poincaré-Lighthill-Kuo Method, Advances in Applied Mechanics, 4, 281-349, Academic Press Inc., New York (1956).
54. Weise, R., Wärmeübergang durch freie Konvektion an quadratischen Platten, Forschung auf dem Gebiete des Ingenieurwesens, 6, 281-292 (1935).
55. Yang, K. T., Possible Similarity Solutions for Laminar Free Convection on Vertical Plates and Cylinders, J. Appl. Mech., Trans. ASME, 82, Ser. E, 230-236 (1960).
56. Yang, K. T., and Jerger, E. W., First-Order Perturbations of Laminar Free-convection Boundary Layers on a Vertical Plate, J. Heat Transfer Trans. ASME, Ser. C, Paper No. 62-WA-187 (to be published).
57. Young, R. J., and Yang, K. T., Effect of Small Cross Flow and Surface-Temperature Variation on Laminar Free Convection along a Vertical Plate, J. Appl. Mech., Trans. ASME, Ser. E, Paper No. 62-WA-81.

## IX. ACKNOWLEDGMENT

This study was performed at Argonne National Laboratory under the AMU program. The authors wish to thank Dr. P. Nelson for his advice and suggestions. Thanks are also due to W. Nico and A. J. Strecok of the Applied Mathematics Division of Argonne for the assistance in the IBM program, and to Jacqueline Lehmann for typing the original manuscript.



ARGONNE NATIONAL LAB WEST



3 4444 00009143 9

X